Circular Motion

A-Level Worksheet

Q1. **(a)** With reference to velocity and acceleration, describe uniform circular motion.

....

....

.... [2]

(b) Two cars are moving around a horizontal circular track. One car follows path X and the other follows path Y, as shown in Fig. 1.1.

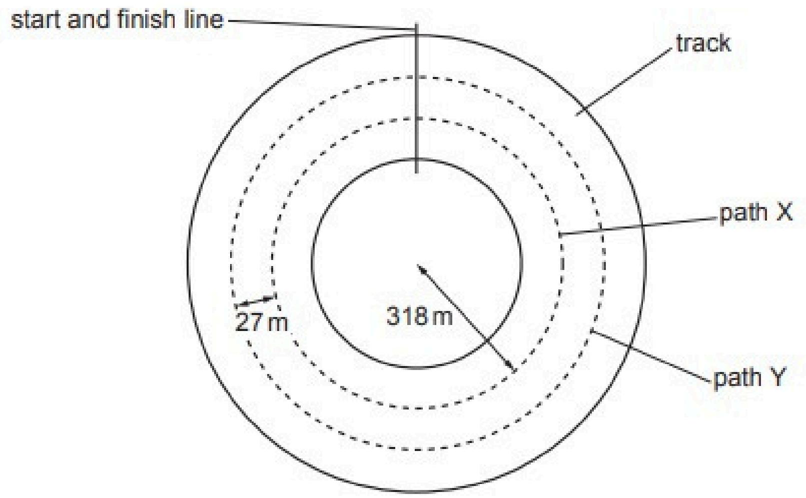

Fig. 1.1 (not to scale)

The radius of path X is 318 m. Path Y is parallel to, and 27 m outside, path X. Both cars have mass 790 kg. The maximum lateral (sideways) friction force *F* that the cars can experience without sliding is the same for both cars.

(i) The maximum speed at which the car on path X can move around the track without sliding is 94 $m\,s^{-1}$.

Calculate *F*.

F = ... N [2]

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(ii) Both cars move around the track. Each car has the maximum speed at which it can move without sliding.

Complete Table 1.1, by placing one tick in each row, to indicate how the quantities indicated for the car on path Y compare with the car on path X.

Table 1.1

	Y less than X	Y same as X	Y greater than X
centripetal acceleration			
maximum speed			
time taken for one lap of the track			

[3]

[Total: 7]

Q2. **(a)** State what is meant by *centripetal* acceleration.

....

....

.... [1]

(b) An unpowered toy car moves freely along a smooth track that is initially horizontal. The track contains a vertical circular loop around which the car travels, as shown in Fig. 1.1.

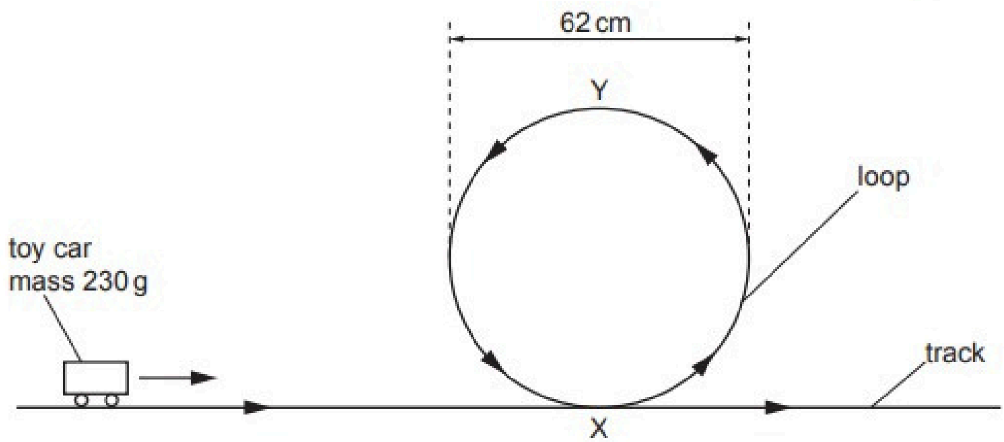

Fig. 1.1

The mass of the car is 230 g and the diameter of the loop is 62 cm. Assume that the resistive forces acting on the car are negligible.

(i) State what happens to the magnitude of the centripetal acceleration of the car as it moves around the loop from X to Y.

.... [1]

(ii) Explain, if the car remains in contact with the track, why the centripetal acceleration of the car at point Y must be greater than $9.8\,m\,s^{-2}$.

....

....

.... [2]

(c) The initial speed at which the car in **(b)** moves along the track is $3.8\,m\,s^{-1}$.

Determine whether the car is in contact with the track at point Y. Show your working.

[3]

(d) Suggest, with a reason but without calculation, whether your conclusion in **(c)** would be different for a car of mass 460 g moving with the same initial speed.

....

....

.... [1]

[Total: 8]

Q3. **(a)** Define the *radian*.

....

....

.... [2]

(b) A telescope gives a clear view of a distant object when the angular displacement between the edges of the object is at least 9.7×10^{-6} rad.

(i) The Moon is approximately 3.8×10^{5} km from Earth.
Estimate the minimum diameter of a circular crater on the Moon's surface that can be seen using the telescope.

diameter = ... km [2]

(ii) Suggest why craters of the same diameter as that calculated in **(i)** but on the surface of Mars are not visible using this telescope.

....

....

.... [2]

Q4. A large bowl is made from part of a hollow sphere.

A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius, as shown in Fig. 2.1.

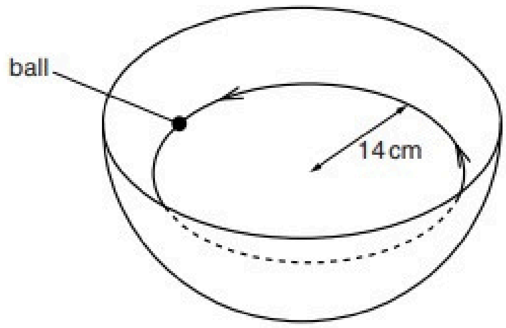

Fig. 2.1

The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball, as shown in Fig. 2.2.

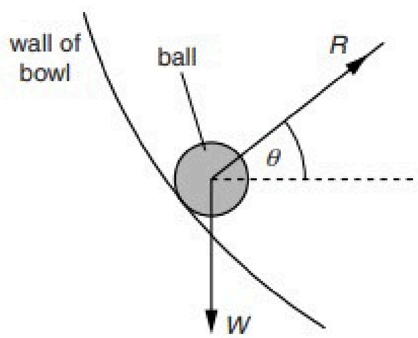

Fig. 2.2

The normal reaction force R is at an angle θ to the horizontal.

(a) **(i)** By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by the expression

$$W = F \tan \theta.$$

[2]

(ii) State the significance of the force *F* for the motion of the ball in the bowl.

....

.... [1]

(b) The ball moves in a circular path of radius 14 cm. For this radius, the angle θ is 28°.

Calculate the speed of the ball.

speed = ... $m\,s^{-1}$ [3]

Q5. **(a)** Define the *radian*.

....

....

.... [2]

(b) A stone of weight 3.0 N is fixed, using glue, to one end P of a rigid rod CP, as shown in Fig. 1.1.

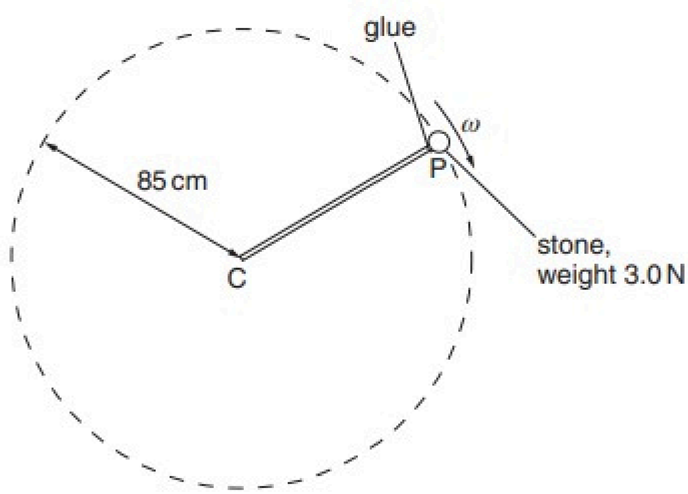

Fig. 1.1

The rod is rotated about end C so that the stone moves in a vertical circle of radius 85 cm.
The angular speed ω of the rod and stone is gradually increased from zero until the glue snaps. The glue fixing the stone snaps when the tension in it is 18 N.

For the position of the stone at which the glue snaps,

(i) on the dotted circle of Fig. 1.1, mark with the letter S the position of the stone, [1]

(ii) calculate the angular speed ω of the stone.

angular speed = ... rad s^{-1}

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Q6. A vertical peg is attached to the edge of a horizontal disc of radius r, as shown in Fig. 4.1.

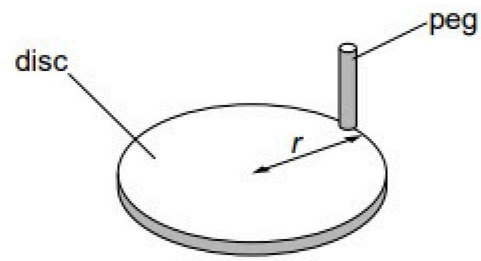

Fig. 4.1

The disc rotates at constant angular speed ω. A horizontal beam of parallel light produces a shadow of the peg on a screen, as shown in Fig. 4.2.

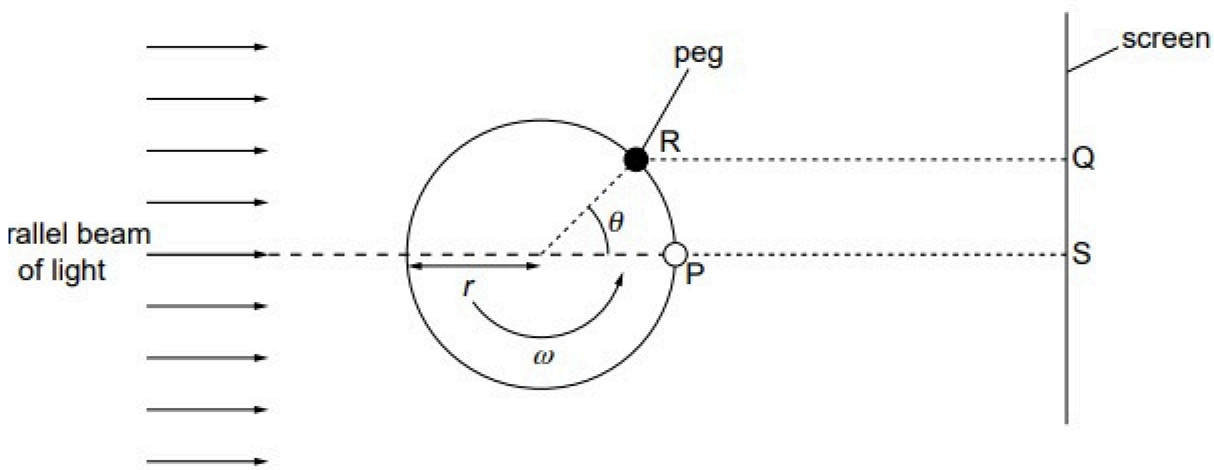

Fig. 4.2 (plan view)

At time zero, the peg is at P, producing a shadow on the screen at S.
At time t, the disc has rotated through angle θ. The peg is now at R, producing a shadow at Q.

(a) Determine,

(i) in terms of ω and t, the angle θ,

.... [1]

(ii) in terms of ω, t and r, the distance SQ.

.... [1]

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(b) Use your answer to **(a)(ii)** to show that the shadow on the screen performs simple harmonic motion.

....

....

.... [2]

(c) The disc has radius r of 12 cm and is rotating with angular speed ω of 4.7 rad s^{-1}.

Determine, for the shadow on the screen,

(i) the frequency of oscillation,

frequency = ... Hz [2]

(ii) its maximum speed.

speed = ... cm s^{-1} [2]

Q7. **(a)** **(i)** Define the *radian*.

....

....

.... [2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

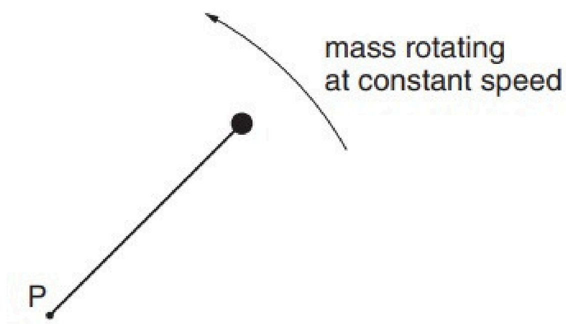

Fig. 1.1

Explain what is meant by the *angular* speed about point P of the mass.

....

....

.... [2]

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

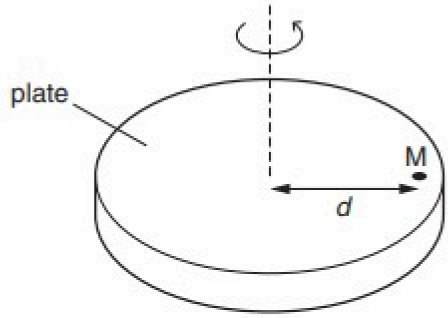

Fig. 1.2

A small mass M is placed on the plate, a distance *d* from the axis of rotation.
The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force *F* between the plate and the mass is given by the expression

$$F = 0.72W,$$

where *W* is the weight of the mass M.
The distance *d* is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working.

number = ...[5]

(c) The plate in **(b)** is covered, when stationary, with mud.
Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

....

....

....[2]

Q8. **(a)** Explain

(i) what is meant by a *radian*,

....

....

.... [2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad.

....

.... [1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 1.1.

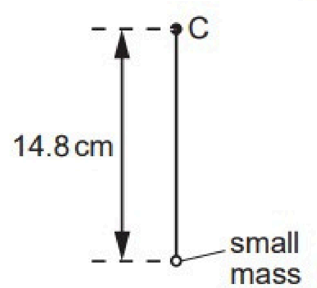

Fig. 1.1

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 1.2.

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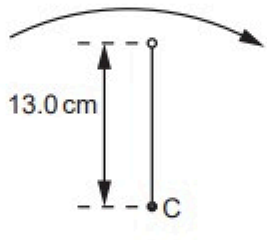

Fig. 1.2

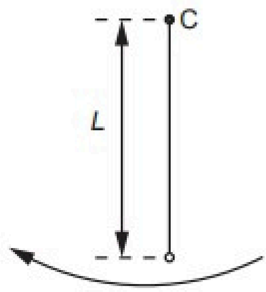

Fig. 1.3

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1}.

[2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 1.3.

Calculate the length L of the cord, assuming it obeys Hooke's law.

L = ... cm [4]

Q9 A steel sphere of mass 0.29 kg is suspended in equilibrium from a vertical spring. The centre of the sphere is 8.5 cm from the top of the spring, as shown in Fig. 2.1.

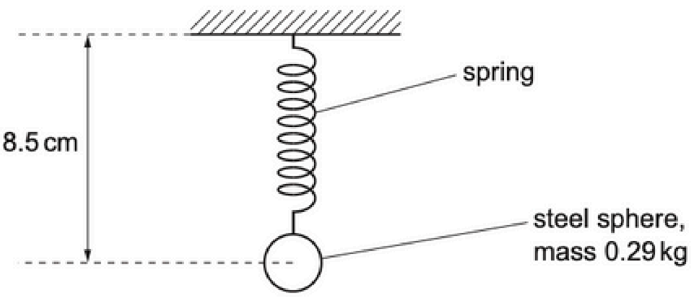

Fig. 2.1

The sphere is now set in motion so that it is moving in a horizontal circle at constant speed, as shown in Fig. 2.2.

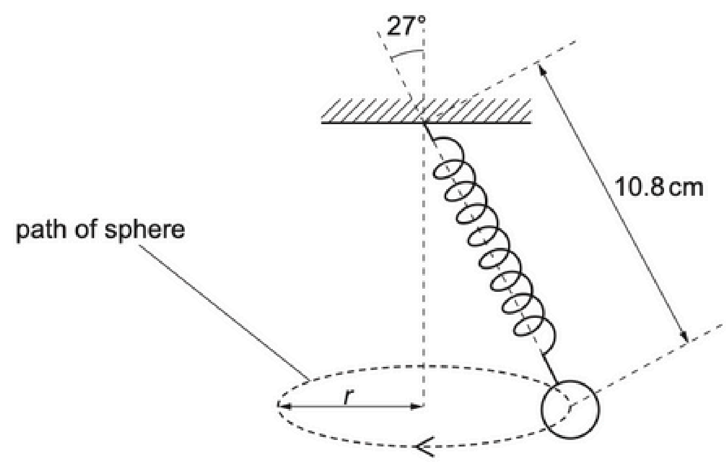

Fig. 2.2

The distance from the centre of the sphere to the top of the spring is now 10.8 cm.

(a) Explain, with reference to the forces acting on the sphere, why the length of the spring in Fig. 2.2 is greater than in Fig. 2.1.

....

....

....

....

.... [3]

(b) The angle between the linear axis of the spring and the vertical is 27°.

(i) Show that the radius r of the circle is 4.9 cm.

[1]

(ii) Show that the tension in the spring is 3.2 N.

[2]

(iii) The spring obeys Hooke's law.

Calculate the spring constant, in $N\,cm^{-1}$, of the spring.

spring constant = ... $N\,cm^{-1}$ [2]

(c) **(i)** Use the information in **(b)** to determine the centripetal acceleration of the sphere.

centripetal acceleration = ... $m\,s^{-2}$ [2]

(ii) Calculate the period of the circular motion of the sphere.

period = ... s [2]

[Total: 12]

Q10 a) Define the radian.

....

.... [1]

(b) The minute hand of a clock revolves at constant angular speed around the face of the clock, completing one revolution every hour. A small piece of modelling clay is attached to the hand with its centre of gravity at a distance L from the fixed end of the hand, as shown in Fig. 1.1.

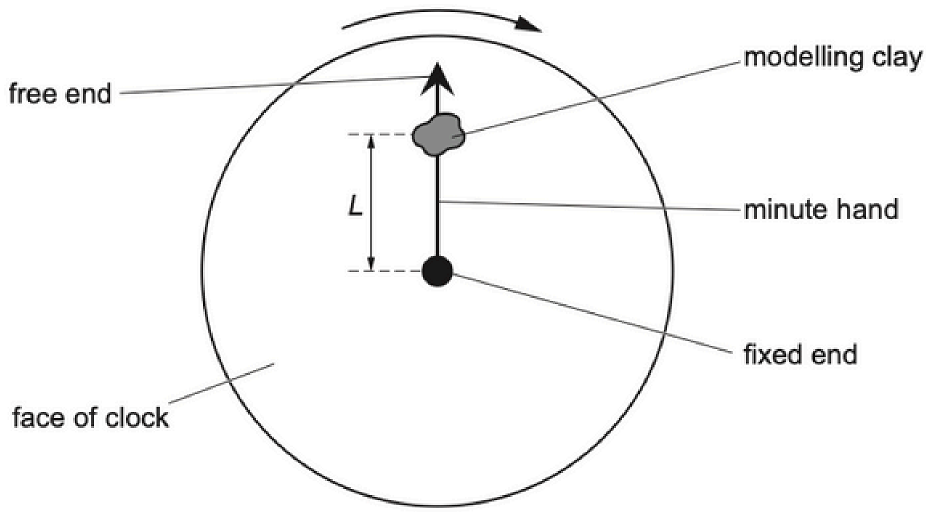

Fig. 1.1

Calculate the angular speed ω of the minute hand.

ω = ... rad s^{-1} [2]

(c) During a time interval of 1400 s, the centre of gravity of the piece of modelling clay in Fig. 1.1 moves through a total distance of 0.44 m.

(i) Calculate the angle through which the minute hand moves in this time interval.

angle = ... rad [1]

(ii) Determine distance *L*.

L = ... m [2]

(iii) Calculate the magnitude of the centripetal acceleration of the piece of modelling clay.

centripetal acceleration = ... $m\,s^{-2}$ [2]

(d) Use your answer in **(c)(iii)** to explain why the variation with time of the magnitude of the force exerted by the minute hand on the piece of modelling clay is negligible as the minute hand undergoes one full revolution.

....

....

.... [2]

[Total: 10]

Mark scheme

Q1.

Question	Answer	Marks
1(a)	constant speed **or** constant magnitude of velocity	B1
	acceleration (always) perpendicular to velocity	B1
1(b)(i)	$F = mv^2 / r$ **or** $v = r\omega$ **and** $F = mr\omega^2$	C1
	$F = 790 \times 94^2 / 318$ $= 22000$ N	A1
1(b)(ii)	centripetal acceleration: same	B1
	maximum speed: greater	B1
	time taken for one lap of the track: greater	B1

Q2.

Question	Answer	Marks
1(a)	acceleration perpendicular to velocity	B1
1(b)(i)	decreases	B1
1(b)(ii)	(acceleration of) $9.8\,\text{m s}^{-2}$ is caused by weight of car **or** centripetal force must be greater than weight of car	B1
	(acceleration > $9.8\,\text{m s}^{-2}$) requires contact <u>force</u> from track **or** (centripetal force > weight) requires contact <u>force</u> from track	B1
1(c)	$\frac{1}{2}mv_Y^2 = \frac{1}{2}mv_X^2 - mgh$	C1
	$a = v^2 / r$	C1
	$v_Y^2 = 3.8^2 - 2 \times 9.81 \times 0.62$ so $v_Y = 1.5\,\text{m s}^{-1}$ $a = 1.5^2 / 0.31 = 7.3\,\text{m s}^{-2}$ (which is less than $9.8\,\text{m s}^{-2}$) so no	A1
	or	
	$v_Y = \sqrt{(9.81 \times 0.31)} = 1.74\,\text{m s}^{-1}$ so $v_X^2 = 1.74^2 + 2 \times 9.81 \times 0.62$ $v_X = 3.9\,\text{m s}^{-1}$ (which is greater than $3.8\,\text{m s}^{-1}$) so no	(A1)
1(d)	acceleration is independent of mass so makes no difference **or** mass cancels in the equation so makes no difference	B1

Q3. **(a)** angle subtended at the centre of a circle — B1
by an arc equal in length to the radius — B1 [2]

(b) **(i)** arc = distance × angle — C1
diameter $= 3.8 \times 10^5 \times 9.7 \times 10^{-6}$
$= 3.7$ km — A1 [2]

(ii) Mars is (much) further from Earth/away (*answer must be comparative*) — B1
angle (at telescope is much) smaller — B1 [2]

Q4. **(a)** **(i)** $F = R\cos\theta$ M1
$W = R\sin\theta$ M1
dividing, $W = F\tan\theta$ A0 [2]
(max. 1 if derivation to final line not shown)

(ii) provides the centripetal force B1 [1]

(b) *either* $F = mv^2/r$ and $W = mg$
or $v^2 = rg/\tan\theta$ C1
$v^2 = (14 \times 10^{-2} \times 9.8)/\tan 28°$ C1
$= 2.58$
$v = 1.6\,\text{m s}^{-1}$ A1 [3]

Q5. **(a)** angle (subtended) at centre of circle B1
(by) arc equal in length to radius B1 [2]

(b) **(i)** point S shown below C B1 [1]

(ii) (max) force / tension = weight + centripetal force C1
centripetal force = $mr\omega^2$ C1
$15 = 3.0/9.8 \times 0.85 \times \omega^2$ C1
$\omega = 7.6\ \text{rad s}^{-1}$ A1 [4]

Q6. **(a)** **(i)** (θ =) ωt *(allow any subject if all terms given)* B1 [1]

(ii) (SQ =) $r\sin\omega t$ *(allow any subject if all terms given)* B1 [1]

(b) this is the solution of the equation $a = -\omega^2 x$ M1
$a = -\omega^2 x$ is the (defining) equation of s.h.m. A1 [2]

(c) **(i)** $f = \omega / 2\pi$ C1
$= 4.7 / 2\pi$
$= 0.75$ Hz A1 [2]

(ii) $v = r\omega$ (r must be identified) C1
$= 4.7 \times 12$
$= 56\,\text{cm s}^{-1}$ A1 [2]

Q7. (a) (i) angle (subtended) at centre of circle — B1
by an arc equal in length to the radius (of the circle) — B1 [2]

(ii) angle swept out per unit time / rate of change of angle — M1
by the string — A1 [2]

(b) friction provides / equals the centripetal force — B1
$0.72\,W = md\omega^2$ — C1
$0.72\,mg = m \times 0.35\omega^2$
$\omega = 4.49$ (rad s^{-1}) — C1
$n = (\omega/2\pi) \times 60$ — B1
$= 43$ min^{-1} (allow 42) — A1 [5]

(c) *either* centripetal force increases as r increases
or centripetal force larger at edge — M1
so flies off at edge first — A1 [2]
($F = mr\omega^2$ so edge first – treat as special case and allow one mark)

Q8. (a) (i) angle subtended at centre of circle B1
arc equal in length to the radius .... B1 [2]

(ii) arc = $r\theta$ and for one revolution, arc = $2\pi r$ M1
so, $\theta = 2\pi r/r = 2\pi$ .... A0 [1]

(b) (i) *either* weight provides/equals the centripetal force
or acceleration of free fall is centripetal acceleration B1
$9.8 = 0.13 \times \omega^2$ M1
$\omega = 8.7$ rad s^{-1} .... A0 [2]

(ii) force in cord = weight + centripetal force (*can be an equation*) C1
force in cord = $(L - 13) \times 5/1.8$ *or* force constant = 5.0/1.8 .... C1
$(L - 13) \times 5/1.8 = 5.0 + 5/9.8 \times L \times 10^{-2} \times 8.7^2$ C1
$L = 17.2$ cm .... A1 [4]
(*constant centripetal force of 5.0 N gives L = 16.6 cm allow 2/4*)

Q9.

(a)	horizontal force on sphere causes centripetal acceleration	**B1**
	weight of sphere is (now) equal to vertical component of tension **or** horizontal and vertical components (of force) (now) combine to give greater tension (in spring)	**B1**
	greater tension in spring so greater extension of spring	**B1**
2(b)(i)	$r = 10.8 \times \sin 27° = 4.9$ cm	**A1**
2(b)(ii)	$T \cos \theta = mg$ **or** $T \cos \theta = W$ **and** $W = mg$	**C1**
	$T \cos 27° = 0.29 \times 9.81$ leading to $T = 3.2$ N	**A1**
2(b)(iii)	$\Delta T = 3.2 - (0.29 \times 9.81)$	**C1**
	$k = \Delta T / \Delta x$ $= [3.2 - (0.29 \times 9.81)] / [10.8 - 8.5]$ $= 0.15$ N cm^{-1}	**A1**
2(c)(i)	centripetal acceleration $= (T \sin \theta) / m$ $= (3.2 \times \sin 27°) / 0.29$	**C1**
	$= 5.0$ m s^{-2}	**A1**
2(c)(ii)	$a = r\omega^2$ **and** $\omega = 2\pi / T$ **or** $a = v^2 / r$ **and** $v = 2\pi r / T$	**C1**
	$T = 2\pi \times \sqrt{(0.049 / 5.0)}$ $= 0.62$ s	**A1**

Q10 a)	angle (subtended at centre of circle) when arc length = radius	B1
1(b)	$\omega = 2\pi / T$	C1
	$= 2\pi / (1.0 \times 60 \times 60)$ $= 1.7 \times 10^{-3}$ rad s^{-1}	A1
1(c)(i)	angle $= 1.7 \times 10^{-3} \times 1400$ $= 2.4$ rad	A1
1(c)(ii)	L = arc length / angle $= 0.44 / 2.4$ **or** $L = 0.44 \times (3600 / 1400) / 2\pi$	C1
	$L = 0.18$ m	A1
1(c)(iii)	$a = r\omega^2$	C1
	$= 0.18 \times (1.745 \times 10^{-3})^2$ $= 5.5 \times 10^{-7}$ m s^{-2}	A1
1(d)	centripetal acceleration is negligible compared with acceleration of free fall **or** numerical comparison establishing answer to **(c)(iii)** $\ll 9.81$	B1
	resultant force is negligible compared with weight (of modelling clay) (so variation is negligible) **or** force exerted by minute hand (approximately) equal (and opposite) to weight of modelling clay	B1