Gravitational Fields
A-Level Worksheet

9702/MJ/23/42/Q1

1 **(a)** State Newton's law of gravitation.

....

....

.... [2]

(b) A satellite is in a circular orbit around a planet. The radius of the orbit is R and the period of the orbit is T. The planet is a uniform sphere.

Use Newton's law of gravitation to show that R and T are related by

$$4\pi^2 R^3 = GMT^2$$

where M is the mass of the planet and G is the gravitational constant.

[2]

(c) The Earth may be considered to be a uniform sphere of mass 5.98×10^{24} kg and radius 6.37×10^6 m.

A geostationary satellite is in orbit around the Earth.

Use the expression in **(b)** to determine the height of the satellite above the Earth's surface.

height = ... m [3]

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(d) Another satellite is in a circular orbit around the Earth with the same orbital radius and period as the satellite in **(c)**.

(i) Calculate the angular speed of the satellite in this orbit. Give a unit with your answer.

angular speed = ... unit .... [2]

(ii) Despite having the same orbital period, the orbit of this satellite is not geostationary.

Suggest **two** ways in which the orbit of this satellite could be different from the orbit of the satellite in **(c)**.

1 ....

....

2 ....

....

[2]

[Total: 11]

2 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) Artemis is a spherical planet that may be assumed to be isolated in space. The variation with distance x from the centre of Artemis of the gravitational potential ϕ is shown in Fig. 1.1.

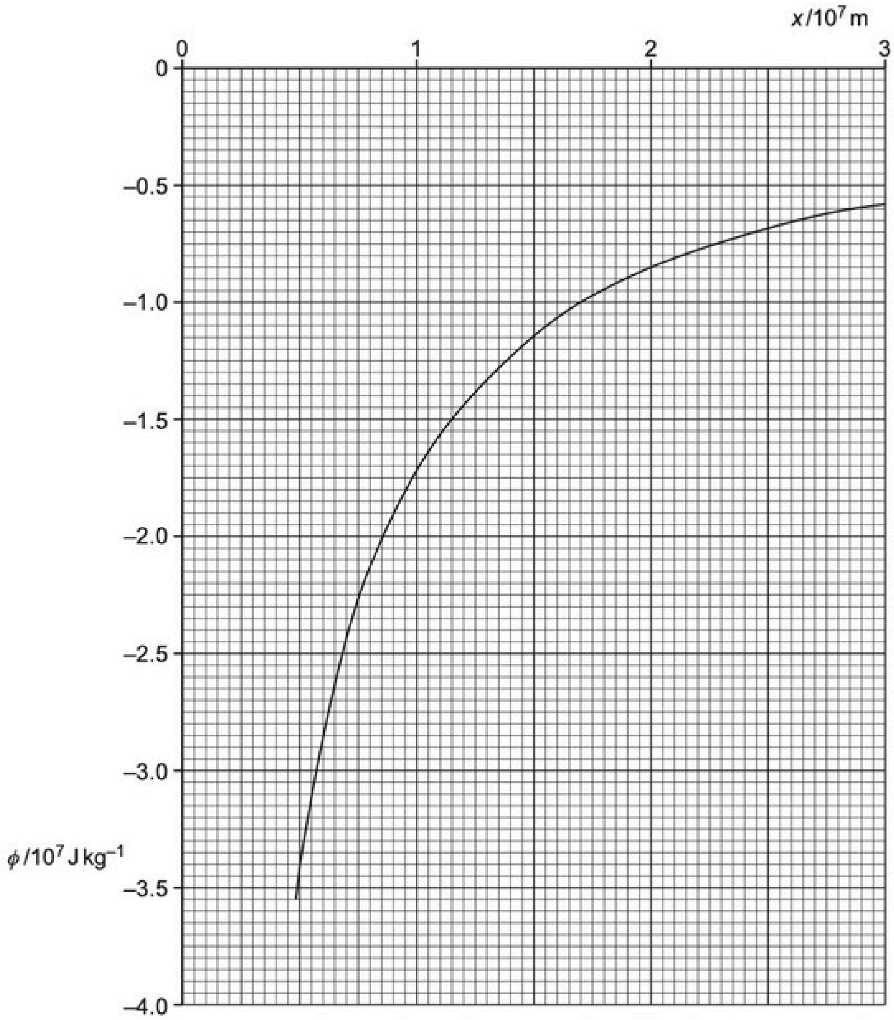

Fig. 1.1

(i) The radius of Artemis is 4800 km.

Determine the value of ϕ on the surface of Artemis.

ϕ = ... $\mathrm{J\,kg^{-1}}$ [1]

(ii) Show that the mass of Artemis is 2.55×10^{24} kg.

[1]

(iii) Calculate the gravitational field strength g on the surface of Artemis.

g = ... $\mathrm{N\,kg^{-1}}$ [2]

(iv) A satellite is in an orbit at a fixed position above a point on the surface of Artemis. The satellite is located above the equator of Artemis at a height above the surface where the gravitational potential is $-0.65 \times 10^{7}\,\mathrm{J\,kg^{-1}}$.

Calculate the period, in hours, of rotation of Artemis.

period = ... hours [4]

(c) State **one** similarity and **one** difference between gravitational potential due to a point mass and electric potential due to a point charge.

similarity ....

....

difference ....

....

[2]

[Total: 12]

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3 **(a)** Define gravitational field. 9702/ON/22/42/Q1

....

.... [1]

(b) A spherical planet can be considered as a point mass at its centre.

(i) On Fig. 1.1, draw gravitational field lines outside the planet to represent the gravitational field due to the planet.

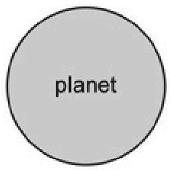

Fig. 1.1

[2]

(ii) A satellite is in a circular orbit around the planet.

Explain, with reference to your answer in **(b)(i)**, why the path of the satellite is circular.

....

....

.... [2]

(c) An object rests on the surface of the Earth at the Equator.
The radius of the Earth is 6.4×10^6 m.

(i) Determine the centripetal acceleration of the object.

centripetal acceleration = ... ms^{-2} [3]

(ii) Describe how the two forces acting on the object give rise to this centripetal acceleration.
You may draw a diagram if you wish.

....

....

.... [2]

[Total: 10]

(a) **(i)** Define gravitational potential at a point.

....

....

.... [2]

(ii) Starting from the equation for the gravitational potential due to a point mass, show that the gravitational potential energy E_P of a point mass m at a distance r from another point mass M is given by

$$E_P = -\frac{GMm}{r}$$

where G is the gravitational constant.

[1]

(b) Fig. 1.1 shows the path of a comet of mass 2.20×10^{14} kg as it passes around a star of mass 1.99×10^{30} kg.

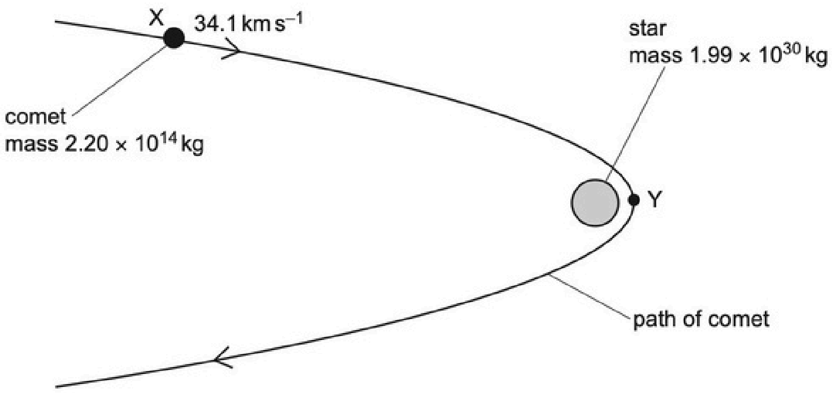

Fig. 1.1 (not to scale)

At point X, the comet is 8.44×10^{11} m from the centre of the star and is moving at a speed of $34.1\,\text{km s}^{-1}$.

At point Y, the comet passes its point of closest approach to the star. At this point, the comet is a distance of 6.38×10^{10} m from the centre of the star.

Both the comet and the star can be considered as point masses at their centres.

(i) Calculate the magnitude of the change in the gravitational potential energy ΔE_P of the comet as it moves from position X to position Y.

ΔE_P = ... J [2]

(ii) State, with a reason, whether the change in gravitational potential energy in **(b)(i)** is an increase or a decrease.

....

.... [1]

(iii) Use your answer in **(b)(i)** to determine the speed, in $km\,s^{-1}$, of the comet at point Y.

speed = ... $km\,s^{-1}$ [3]

(c) A second comet passes point X with the same speed as the comet in **(b)** and travelling in the same direction. This comet is gradually losing mass. The mass of this comet when it passes point X is the same as the mass of the comet in **(b)**.

Suggest, with a reason, how the path of the second comet compares with the path shown in Fig. 1.1.

....

.... [1]

[Total: 10]

5 **(a)** **(i)** State Newton's law of gravitation.

....

....

.... [2]

(ii) Use Newton's law of gravitation to show that the gravitational field strength g at a distance r away from a point mass M is given by

$$g = \frac{GM}{r^2}.$$

[2]

(b) The Earth has a mass of 5.98×10^{24} kg and a radius of 6.37×10^{6} m.
The Moon has a mass of 7.35×10^{22} kg and a radius of 1.74×10^{6} m.
The Earth and the Moon can both be considered as point masses at their centres. Their centres are a distance of 3.84×10^{8} m apart.

(i) Show that the gravitational field strength at the surface of the Moon due to the mass of the Moon is $1.62\,N\,kg^{-1}$.

[1]

(ii) Explain why there is a point X on the line between the centres of the Earth and the Moon where the resultant gravitational field strength due to the Earth and the Moon is zero.

....

....

.... [2]

(iii) Calculate the distance *x* of point X from the centre of the Moon.

x = ... m [3]

[Total: 10]

6 **(a)** The point P in Fig. 1.1 represents a point mass. 9702/FM/22/42/Q1

On Fig. 1.1, draw lines to represent the gravitational field around P.

Fig. 1.1

[2]

(b) A moon is in circular orbit around a planet.

Explain why the path of the moon is circular.

....

....

....

.... [2]

(c) Many moons are in circular orbit about a planet.

The angular velocity of a moon is ω when the orbit of the moon has a radius r about the planet.

Fig. 1.2 shows the variation of r^3 with $1/\omega^2$ for these moons.

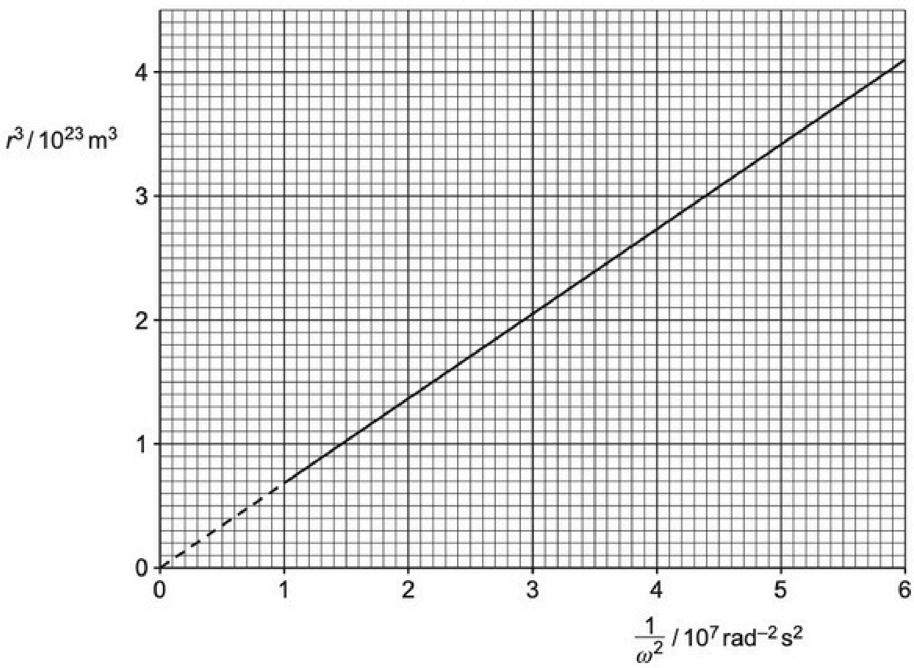

Fig. 1.2

(i) Show that the mass M of the planet is given by the expression

$$M = \frac{\text{gradient}}{G}$$

where G is the gravitational constant.

[2]

(ii) Use Fig. 1.2 and the expression in **(c)(i)** to show that the mass *M* of the planet is 1.0×10^{26} kg.

[1]

(iii) Determine the speed of a moon in orbit around the planet with an orbital radius of 1.2×10^{8} m.

speed = ... $m\,s^{-1}$ [3]

[Total: 10]

7 **(a)** Define *gravitational potential*.

....

....

.... [2]

(b) The Earth E and the Moon M can both be considered as isolated point masses at their centres. The mass of the Earth is 5.98×10^{24} kg and the mass of the Moon is 7.35×10^{22} kg. The Earth and the Moon are separated by a distance of 3.84×10^{8} m, as shown in Fig. 2.1.

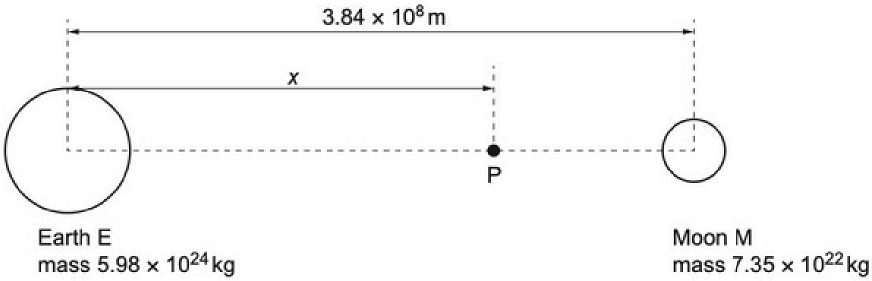

Fig. 2.1 (not to scale)

P is a point, on the line joining the centres of E and M, where the resultant gravitational field strength is zero. Point P is at a distance *x* from the centre of the Earth.

(i) Explain how it is possible for the gravitational field strength to be zero despite the presence of two large masses nearby.

....

....

.... [2]

(ii) Show that *x* is approximately 3.5×10^{8} m.

[2]

(iii) Calculate the gravitational potential ϕ at point P.

ϕ = ... $J\,kg^{-1}$ [3]

[Total: 9]

8 The Earth may be assumed to be an isolated uniform sphere with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 1200 kg is in a circular orbit about the Earth in the Earth's gravitational field. The period of the orbit is 94 minutes.

(a) Define *gravitational field strength*.

....

.... [1]

(b) Calculate the radius of the orbit of the satellite.

radius = ... m [3]

(c) Rockets on the satellite are fired so that the satellite enters a different circular orbit that has a period of 150 minutes. The change in the mass of the satellite may be assumed to be negligible.

(i) Show that the radius of the new orbit is 9.4×10^{6} m.

[2]

(ii) State, with a reason, whether the gravitational potential energy of the satellite increases or decreases.

....

.... [1]

(iii) Determine the magnitude of the change in the gravitational potential energy of the satellite.

change in potential energy = ... J [3]

[Total: 10]

9 **(a)** Define *gravitational field strength*. 9702/MJ/21/42/Q1

....

.... [1]

(b) An isolated planet is a uniform sphere of radius 3.39×10^6 m. Its mass of 6.42×10^{23} kg may be considered to be a point mass concentrated at its centre. The planet rotates about its axis with a period of 24.6 hours.

For an object resting on the surface of the planet at the equator, calculate, to three significant figures:

(i) the gravitational field strength

field strength = ... N kg^{-1} [2]

(ii) the centripetal acceleration

acceleration = ... m s^{-2} [2]

(iii) the force per unit mass exerted on the object by the surface of the planet.

force per unit mass = ... N kg^{-1} [1]

[Total: 6]

10 **(a)** State Newton's law of gravitation. 9702/42/F/M/21/Q1

....

....

.... [2]

(b) Planets have been observed orbiting a star in another solar system. Measurements are made of the orbital radius r and the time period T of each of these planets.

The variation with R^3 of T^2 is shown in Fig. 1.1.

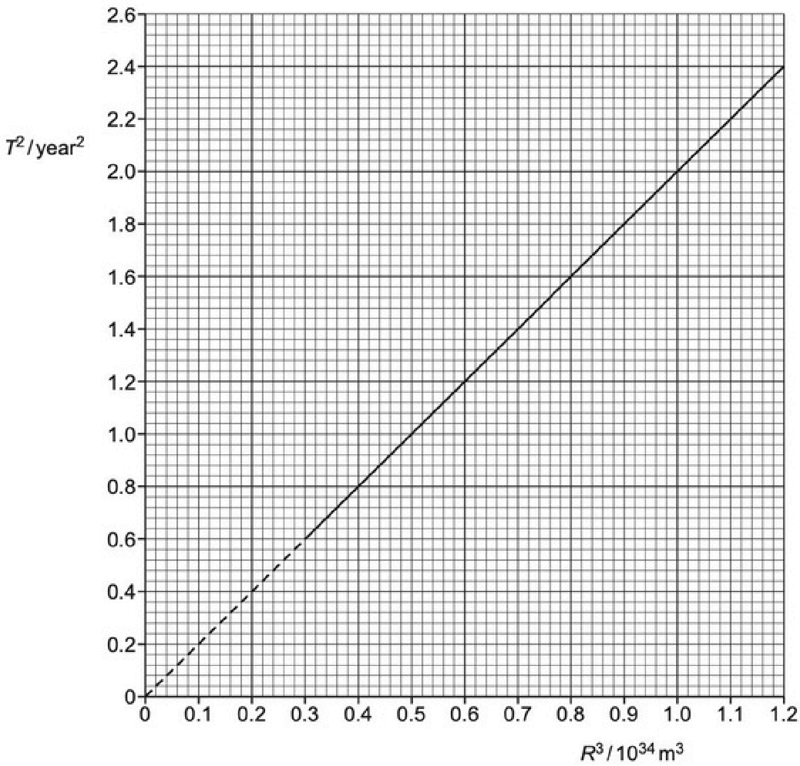

Fig. 1.1

The relationship between T and R is given by

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

where G is the gravitational constant and M is the mass of the star.

Determine the mass M.

M = ... kg [3]

(c) A rock of mass m is also in orbit around the star in **(b)**. The radius of the orbit is r.

(i) Explain why the gravitational potential energy of the rock is negative.

....

....

....

.... [3]

(ii) Show that the kinetic energy E_k of the rock is given by

$$E_k = \frac{GMm}{2r}.$$

[2]

(iii) Use the expression in **(c)(ii)** to derive an expression for the total energy of the rock.

[2]

[Total: 12]

11 **(a)** Define *gravitational potential* at a point. 9702/42/O/N/20/Q1

....

....

.... [2]

(b) The Earth may be considered to be a uniform sphere of radius 6.4×10^6 m with its mass of 6.0×10^{24} kg concentrated at its centre.

A satellite of mass 2.4×10^3 kg is launched from the Equator. It is placed in an equatorial orbit at a height of 5.6×10^6 m above the Earth's surface.

(i) Calculate the change ΔE_P in gravitational potential energy of the satellite for its movement from the surface of the Earth to its position in the equatorial orbit.

ΔE_P = ... J [3]

(ii) Determine the speed of the satellite when in orbit.

speed = ... $m s^{-1}$ [3]

(c) Before the satellite in **(b)** is launched, its speed at the Equator due to the Earth's rotation is $470\,\text{m}\,\text{s}^{-1}$.

Suggest why the energy required to launch the satellite depends on whether the satellite, in its orbit, is travelling from west to east or from east to west.

....

.... [1]

[Total: 9]

12 **(a)** **(i)** State what is meant by a *field of force*. **9702/41/O/N/20/Q1**

....

....

.... [2]

(ii) Define *gravitational field strength*.

....

.... [1]

(b) An isolated planet may be assumed to be a uniform sphere of radius $3.39 \times 10^{6}\,\text{m}$ with its mass of $6.42 \times 10^{23}\,\text{kg}$ concentrated at its centre.

Calculate the gravitational field strength at the surface of the planet.

field strength = ... $\text{N}\,\text{kg}^{-1}$ [3]

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(c) Calculate the height above the surface of the planet in **(b)** at which the gravitational field strength is 1.0% less than its value at the surface of the planet.

height = ... m [3]

[Total: 9]

13 **(a)** Define *gravitational potential* at a point. **9702/42/M/J/20/Q1**

....

....

.... [2]

(b) An isolated solid sphere of radius r may be assumed to have its mass M concentrated at its centre. The magnitude of the gravitational potential at the surface of the sphere is ϕ.

On Fig. 1.1, show the variation of the gravitational potential with distance d from the centre of the sphere for values of d from $d = r$ to $d = 4r$.

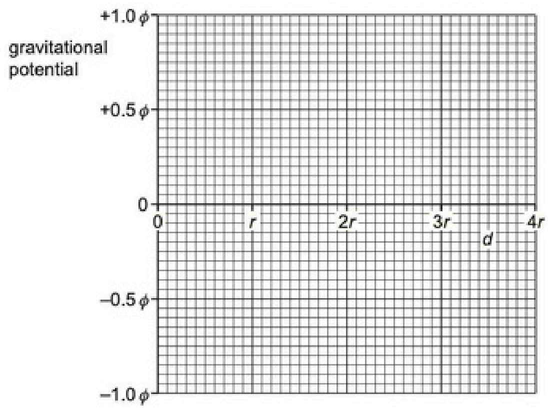

Fig. 1.1 [3]

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(c) The sphere in **(b)** is a planet with radius *r* of 6.4×10^6 m and mass *M* of 6.0×10^{24} kg. The planet has no atmosphere.

A rock of mass 3.4×10^3 kg moves directly towards the planet. Its distance from the centre of the planet changes from 4*r* to 3*r*.

(i) Calculate the change in gravitational potential energy of the rock.

change = ... J [3]

(ii) Explain whether the rock's speed increases, decreases or stays the same.

....

.... [2]

[Total: 10]

14 **(a)** Define *gravitational potential* at a point. **9702/42/F/M/20/Q1**

....

....

.... [2]

(b) TESS is a satellite of mass 360 kg in a circular orbit about the Earth as shown in Fig. 1.1.

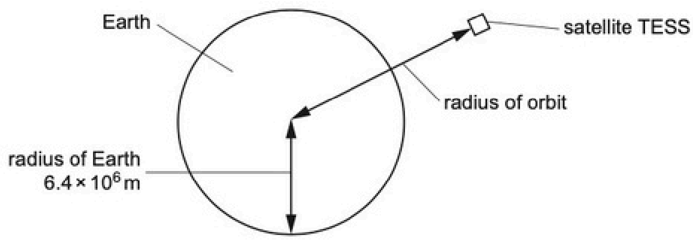

Fig. 1.1 (not to scale)

The radius of the Earth is 6.4×10^6 m and the mass of the Earth, considered to be a point mass at its centre, is 6.0×10^{24} kg.

(i) It takes TESS 13.7 days to orbit the Earth.

Show that the radius of orbit of TESS is 2.4×10^8 m.

[3]

(ii) Calculate the change in gravitational potential energy between TESS in orbit and TESS on a launch pad on the surface of the Earth.

change in gravitational potential energy = ... J [3]

(iii) Use the information in **(b)(i)** to calculate the ratio:

$$\frac{\text{gravitational field strength on surface of Earth}}{\text{gravitational field strength at location of TESS in orbit}} .$$

ratio = ... [2]

[Total: 10]

15 **(a)** State Newton's law of gravitation. **9702/41/O/N/19/Q1**

....

....

.... [2]

(b) A geostationary satellite orbits the Earth. The orbit of the satellite is circular and the period of the orbit is 24 hours.

(i) State **two** other features of this orbit.

1. ....

....

2. ....

....

[2]

(ii) The radius of the orbit of the satellite is 4.23×10^4 km.

Determine a value for the mass of the Earth. Explain your working.

mass = ... kg [4]

[Total: 8]

16 **(a)** Two point masses are separated by a distance *x* in a vacuum. 9702/42/M/J/19/Q1
State an expression for the force *F* between the two masses *M* and *m*.
State the name of any other symbol used.

....

....

....[1]

(b) A small sphere S is attached to one end of a rod, as shown in Fig. 1.1.

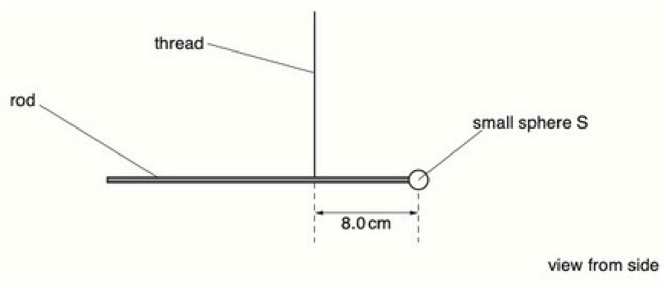

Fig. 1.1 (not to scale)

The rod hangs from a vertical thread and is horizontal.
The distance from the centre of sphere S to the thread is 8.0 cm.

A large sphere L is placed near to sphere S, as shown in Fig. 1.2.

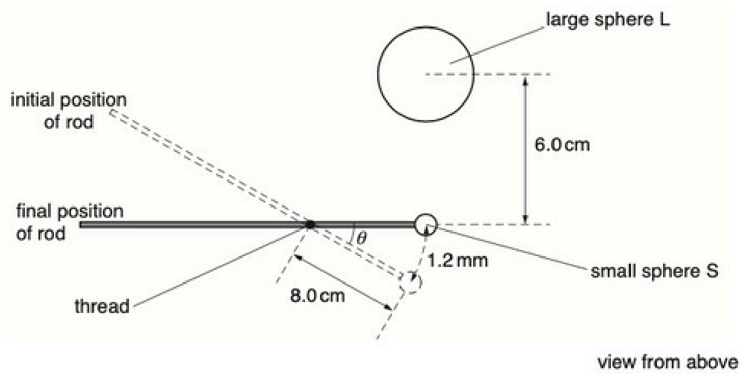

Fig. 1.2 (not to scale)

There is a force of attraction between spheres S and L, causing sphere S to move through a distance of 1.2 mm.
The line joining the centres of S and L is normal to the rod.

(i) Show that the angle θ through which the rod rotates is 1.5×10^{-2} rad.

[1]

(ii) The rotation of the rod causes the thread to twist.
The torque T (in N m) required to twist the thread through an angle β (in rad) is given by

$$T = 9.3 \times 10^{-10} \times \beta.$$

Calculate the torque in the thread when sphere L is positioned as shown in Fig. 1.2.

torque = ... N m [1]

(c) The distance between the centres of spheres S and L is 6.0 cm.
The mass of sphere S is 7.5 g and the mass of sphere L is 1.3 kg.

(i) By equating the torque in **(b)(ii)** to the moment about the thread produced by gravitational attraction between the spheres, calculate a value for the gravitational constant.

gravitational constant = ... N m^2 kg^{-2} [3]

(ii) Suggest why the total force between the spheres may not be equal to the force calculated using Newton's law of gravitation.

....

....[1]

[Total: 7]

17 (a) Two point masses are isolated in space and are separated by a distance x. 9702/43/M/J/19/Q1

State an expression relating the gravitational force F between the two masses to the magnitudes M and m of the masses. State the name of any other symbol used.

....

....

.... [1]

(b) A spacecraft is to be put into a circular orbit about a spherical planet.

The planet may be considered to be isolated in space. The mass of the planet, assumed to be concentrated at its centre, is 7.5×10^{23} kg. The radius of the planet is 3.4×10^{6} m.

(i) The spacecraft is to orbit the planet at a height of 2.4×10^{5} m above the surface of the planet. At this altitude, there is no atmosphere.

Show that the speed of the spacecraft in its orbit is $3.7 \times 10^{3}\,\text{m s}^{-1}$.

[2]

(ii) One possible path of the spacecraft as it approaches the planet is shown in Fig. 1.1.

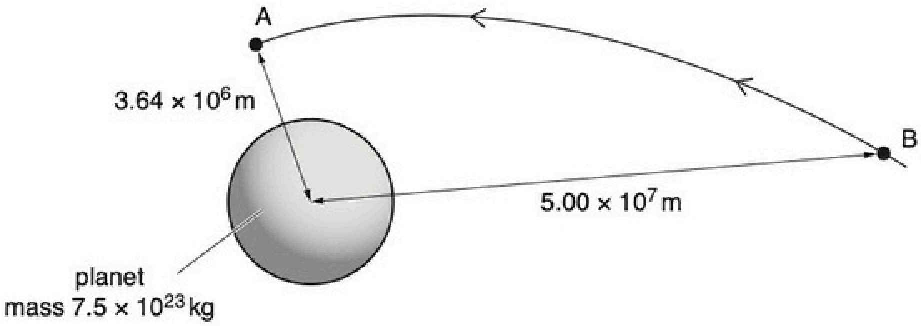

Fig. 1.1 (not to scale)

The spacecraft enters the orbit at point A with speed $3.7 \times 10^3\,\text{m s}^{-1}$.

At point B, a distance of $5.00 \times 10^7\,\text{m}$ from the centre of the planet, the spacecraft has a speed of $4.1 \times 10^3\,\text{m s}^{-1}$. The mass of the spacecraft is 650 kg.

For the spacecraft moving from point B to point A, show that the change in gravitational potential energy of the spacecraft is $8.3 \times 10^9\,\text{J}$.

[3]

(c) By considering changes in gravitational potential energy and in kinetic energy of the spacecraft, determine whether the total energy of the spacecraft increases or decreases in moving from point B to point A. A numerical answer is not required.

....

....

....

.... [2]

[Total: 8]

18 **(a)** **(i)** Define *gravitational potential* at a point. 9702/42/F/M/19/Q1

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....

.... [2]

(ii) Use your answer in **(i)** to explain why the gravitational potential near an isolated mass is always negative.

....

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.... [3]

(b) A spherical planet has mass 6.00×10^{24} kg and radius 6.40×10^{6} m.
The planet may be assumed to be isolated in space with its mass concentrated at its centre.

A satellite of mass 340 kg is in a circular orbit about the planet at a height 9.00×10^{5} m above its surface.

For the satellite:

(i) show that its orbital speed is $7.4 \times 10^{3}\,\text{m}\,\text{s}^{-1}$

[2]

(ii) Calculate its gravitational potential energy.

energy = ... J [3]

(c) Rockets on the satellite are fired for a short time. The satellite's orbit is now closer to the surface of the planet.

State and explain the change, if any, in the kinetic energy of the satellite.

....

....

....

.... [2]

[Total: 12]

19 (a) (i) State what is meant by *gravitational field strength*. 9702/42/O/N/18/Q1

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....

....[1]

(ii) Explain why, at the surface of a planet, gravitational field strength is numerically equal to the acceleration of free fall.

....

....

....[1]

(b) An isolated uniform spherical planet has radius R.
The acceleration of free fall at the surface of the planet is g.

On Fig. 1.1, sketch a graph to show the variation of the acceleration of free fall with distance x from the centre of the planet for values of x in the range $x = R$ to $x = 4R$.

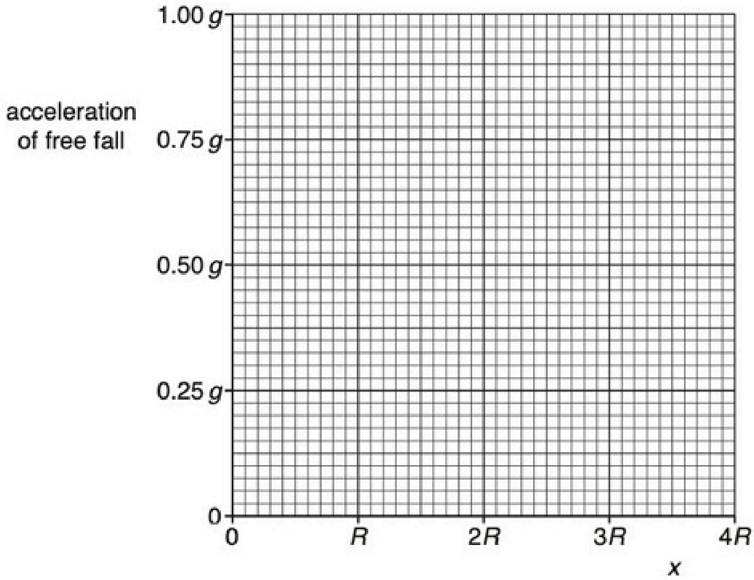


Fig. 1.1

[3]

(c) The planet in **(b)** has radius R equal to 3.4×10^3 km and mean density 4.0×10^3 kg m^{-3}.

Calculate the acceleration of free fall at a height R above its surface.

acceleration of free fall = ... m s^{-2} [3]

[Total: 8]

Q20 **(a)** **(i)** State what is indicated by the direction of the gravitational field line at a point in a gravitational field.

....

.... [1]

(ii) Explain, with reference to gravitational field lines, why the gravitational field near the surface of the Earth is approximately constant for small changes in height.

....

....

.... [2]

(b) A large isolated uniform sphere has mass *M* and radius *R*.

Point P lies on a straight line passing through the centre of the sphere, at a variable displacement *x* from the centre, as shown in Fig. 1.1.

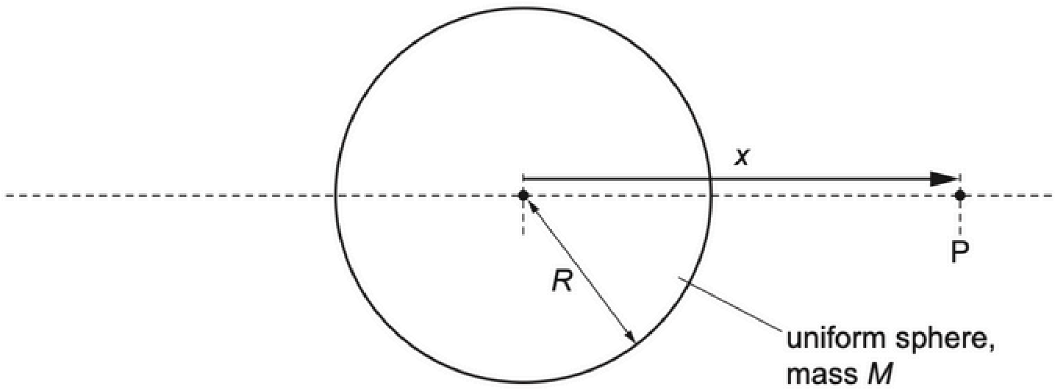


Fig. 1.1

Fig. 1.2 shows the variation with x of the gravitational field g at point P due to the sphere for the values of x for which P is inside the sphere.

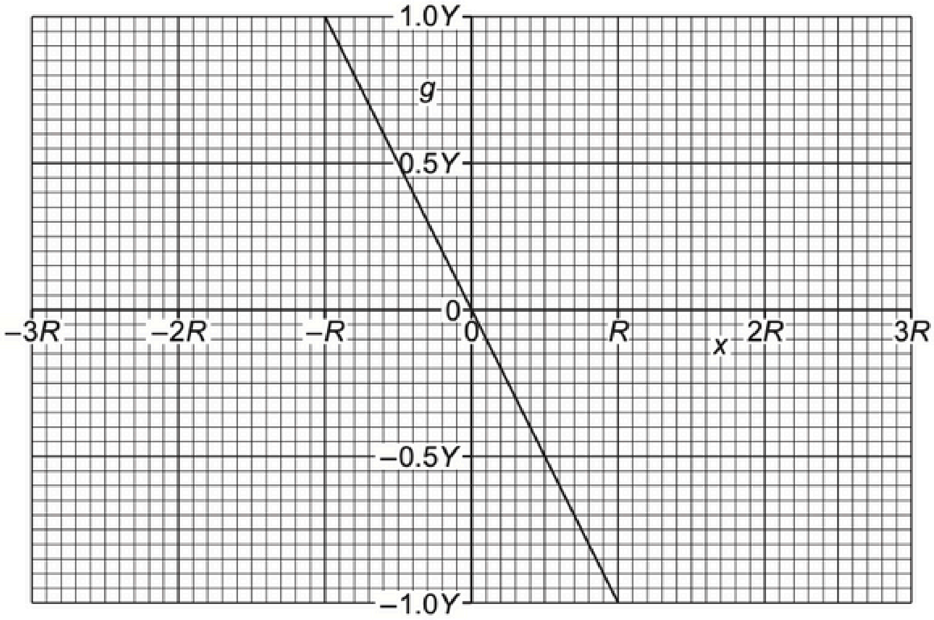


Fig. 1.2

The magnitude of the gravitational field at the surface of the sphere is Y.

(i) Determine an expression for Y in terms of M and R. Identify any other symbols that you use.

[2]

(ii) Explain why, at the surface of the sphere, g always has the opposite sign to x.

....

....

.... [2]

(iii) Complete Fig. 1.2 to show the variation of g with x for values of x, up to $\pm 3R$, for which point P is outside the sphere. [3]

[Total: 10]

Q21 **(a)** **(i)** Define gravitational potential at a point.

....

....

.... [2]

(ii) The Moon may be considered to be an isolated uniform sphere of mass 7.3×10^{22} kg and radius 1.7×10^{6} m.

Calculate the gravitational potential at the surface of the Moon. Give a unit with your answer.

gravitational potential = ... unit .... [2]

(b) An isolated uniform spherical planet has gravitational potential ϕ at its surface.

A particle of mass m is projected vertically upwards from the surface. The particle is given just enough kinetic energy to travel to an infinite distance away from the planet, escaping from the gravitational pull of the planet, without any additional work being done on it.

(i) Determine an expression, in terms of m and ϕ, for the gravitational potential energy E_P of the particle at the surface of the planet.

E_P = ... [1]

(ii) Show that the speed v at which the particle is projected upwards from the surface of the planet is given by

$$v = \sqrt{-2\phi}.$$

[2]

(c) A particle is moving upwards at the surface of the Moon.

Use your answer in **(a)(ii)** and the expression in **(b)(ii)** to determine the minimum speed of this particle that will result in it escaping from the gravitational pull of the Moon.

speed = ... $m s^{-1}$ [1]

9702/42/March 2024

Q22 **(a)** Explain why the gravitational potential near to a point mass is negative.

....

....

....

.... [2]

(b) A planet may be assumed to be a uniform sphere. It has gravitational potential ϕ at distance r from the centre of the planet.

The variation with $\frac{1}{r}$ of ϕ is shown in Fig. 1.1.

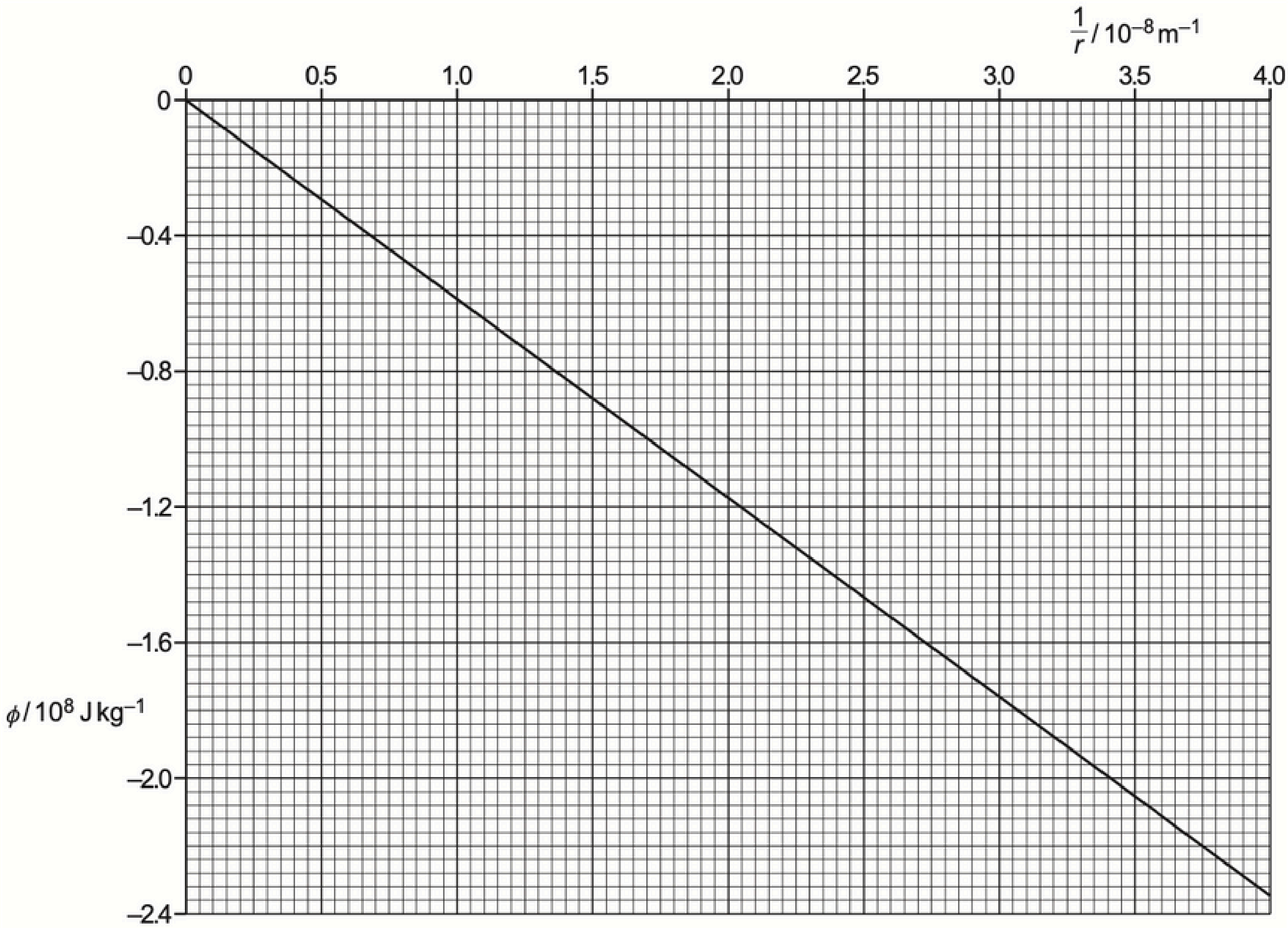


Fig. 1.1

(i) Show that the mass of the planet is 8.8×10^{25} kg.

[2]

(ii) The period of rotation of the planet is 0.72 Earth days.

A satellite in orbit around the planet remains above the same point on the surface of the planet.

Use the mass of the planet in **(b)(i)** to determine the radius *R* of the orbit of the satellite.

R = ... m [3]

(iii) The speed of the satellite in **(b)(ii)** is $8400\,\text{m}\,\text{s}^{-1}$. The mass of the satellite is 1200 kg.

Determine the additional energy required to move the satellite from its orbit to infinity.

energy required = ... J [3]

[Total: 10]

Q23 **(a)** Define gravitational potential at a point.

....

....

.... [2]

(b) A satellite X, of mass M, orbits a planet at a constant distance $4R$ from the centre of the planet, as shown in Fig. 1.1.

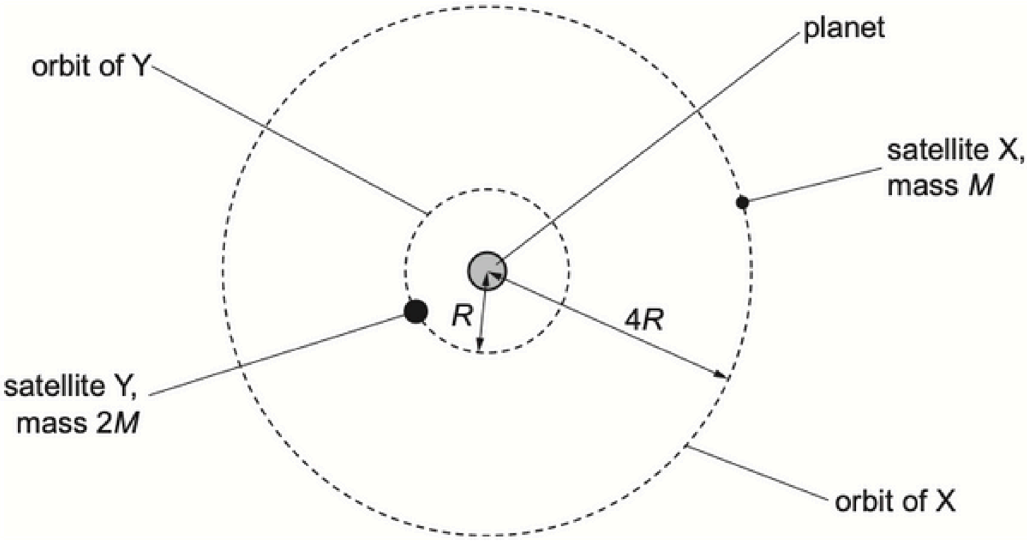


Fig. 1.1 (not to scale)

A second satellite Y, of mass $2M$, orbits the planet with orbital radius R.

The gravitational potential at X due to the planet is $-\Phi$. The planet is a uniform sphere.

(i) Explain why the gravitational potential at X is negative.

....

....

.... [2]

(ii) State an expression, in terms of Φ, for the gravitational potential at Y due to the planet.

gravitational potential = ... [2]

(iii) Complete Table 1.1 by giving expressions, in terms of some or all of *M*, *R* and *Φ*, for the quantities indicated for each of the satellites X and Y.

Table 1.1

	satellite X	satellite Y
gravitational field strength at satellite due to planet		
gravitational potential energy of satellite		

[4]

[Total: 10]

Question	Answer	Marks
1(a)	(gravitational) force is (directly) proportional to product of masses	B1
	force (between point masses) is inversely proportional to the square of their separation	B1
1(b)	$GMm / R^2 = mR\omega^2$	M1
	$\omega = 2\pi / T$ and algebra leading to $4\pi^2R^3 = GMT^2$	A1
	or	
	$GMm / R^2 = mv^2 / R$	(M1)
	$v = 2\pi R / T$ and algebra leading to $4\pi^2R^3 = GMT^2$	(A1)
1(c)	$4\pi^2 \times R^3 = 6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times (24 \times 60 \times 60)^2$ ($R = 4.22 \times 10^7$ m)	C1
	$h = R - (6.37 \times 10^6)$	C1
	$h = (4.22 \times 10^7) - (6.37 \times 10^6)$ $= 3.6 \times 10^7$ m	A1
1(d)(i)	$\omega = 2\pi / T$	C1
	$= 2\pi / (24 \times 60 \times 60)$ $= 7.3 \times 10^{-5}$ rad s^{-1}	A1
1(d)(ii)	orbit is from east to west	B1
	orbit is not equatorial / orbit is polar	B1

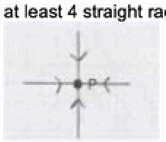
Question	Answer	Marks
2(a)	work done per unit mass	B1
	work (done on mass) moving mass from infinity (to the point)	B1
2(b)(i)	-3.55×10^7 J kg^{-1}	B1
2(b)(ii)	$\phi = -\frac{GM}{r}$ $M = -\frac{-3.55 \times 10^7 \times 4800000}{6.67 \times 10^{-11}}$ $= 2.55 \times 10^{24}$ kg	B1
2(b)(iii)	$g = \frac{GM}{r^2}$ **or** $g = -\frac{\phi}{r}$	C1
	$= \frac{6.67 \times 10^{-11} \times 2.55 \times 10^{24}}{4800000^2}$ **or** $= \frac{3.55 \times 10^7}{4800000}$ $= 7.4$ N kg^{-1}	A1
2(b)(iv)	r in range 2.60×10^7 to 2.65×10^7 m	C1
	$\frac{mv^2}{r} = \frac{GMm}{r^2}$ **and** $v = \frac{2\pi r}{T}$ **or** $mr\omega^2 = \frac{GMm}{r^2}$ **and** $\omega = \frac{2\pi}{T}$	C1
	$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2 \times (2.65 \times 10^7)^3}{6.67 \times 10^{-11} \times 2.55 \times 10^{24}} = 4.20 \times 10^9$	C1
	$T = 64\,800$ s $= 18$ hours	A1

Question	Answer	Marks
2(c)	similarity – any one point from • inversely proportional to distance (from point) • points of equal potential lie on concentric spheres • zero at infinite distance	B1
	difference – any one point from • gravitational potential is (always) negative • electric potential can be positive or negative	B1

Question	Answer	Marks
3(a)	force per unit mass	B1
3(b)(i)	lines drawn are radial from the surface	B1
	arrows show pointing towards planet	B1
3(b)(ii)	field lines show force (on satellite) is towards centre of planet **or** velocity of satellite is perpendicular to field lines	B1
	(gravitational) force perpendicular to velocity causes centripetal acceleration	B1
3(c)(i)	T = 24 hours	C1
	$a = r\omega^2$ **and** $\omega = 2\pi / T$ **or** $a = v^2 / r$ **and** $v = 2\pi r / T$ **or** $a = 4\pi^2 r / T^2$	C1
	$a = (4\pi^2 \times 6.4 \times 10^6) / (24 \times 60 \times 60)^2$ $= 0.034\ \text{m s}^{-2}$	A1
3(c)(ii)	identification of the two forces acting on the object as gravitational force and (normal) contact force	M1
	gravitational force and normal contact force are in opposite directions, and their resultant causes the (centripetal) acceleration	A1

Question	Answer	Marks
4(a)(i)	work (done) per unit mass	B1
	work (done on mass) in moving mass from infinity (to the point)	B1
4(a)(ii)	$E_P = \phi m$ $E_P = (-GM / r) \times m = -GMm / r$ **or** $\phi = -GM / r$ **and** $E_P = \phi m = -GMm / r$	B1
4(b)(i)	$\Delta E_P = 6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 2.20 \times 10^{14} \times [1 / (6.38 \times 10^{10}) - 1 / (8.44 \times 10^{11})]$	C1
	$= 4.23 \times 10^{23}$ J	A1
4(b)(ii)	(gravitational) force is attractive so decrease **or** (gravitational) force does work so decrease	B1
4(b)(iii)	$\Delta E_P = \frac{1}{2}m(v_2^2 - v_1^2)$	C1
	$4.23 \times 10^{23} = \frac{1}{2} \times 2.20 \times 10^{14} \times (v^2 - 34\,100^2)$	C1
	v (= 70 800 m s^{-1}) = 70.8 km s^{-1}	A1
4(c)	both PE and KE equations include m, so path is unchanged	B1

Question	Answer	Marks
5(a)(i)	(gravitational) force is (directly) proportional to product of masses	B1
	force (between point masses) is inversely proportional to the square of their separation	B1
5(a)(ii)	$g = F / m$	C1
	$F = GMm / r^2$ and so $g = [GMm / r^2] / m = GM / r^2$	A1
5(b)(i)	$g = (6.67 \times 10^{-11} \times 7.35 \times 10^{22}) / (1.74 \times 10^6)^2 = 1.62\ \text{N kg}^{-1}$	A1
5(b)(ii)	fields (due to Earth and the Moon) have equal magnitudes	B1
	fields (due to Earth and the Moon) are in opposite directions	B1
5(b)(iii)	distance of X from Earth = $(3.84 \times 10^8 - x)$	C1
	$(G \times)\ 7.35 \times 10^{22} / x^2 = (G \times)\ 5.98 \times 10^{24} / (3.84 \times 10^8 - x)^2$	C1
	$x = 3.8 \times 10^7$ m	A1

Question	Answer	Marks
6(a)	at least 4 straight radial lines to P  	B1
	all arrows pointing along the lines towards P	B1
6(b)	**Any 2 from:** gravitational force provides the centripetal force (centripetal or gravitational) force has constant magnitude (centripetal or gravitational) force is perpendicular to velocity (of moon) / direction of motion (of moon)	B2
6(c)(i)	$\frac{GMm}{r^2} = mr\omega^2$	M1
	$M = \frac{r^3\omega^2}{G}$ and gradient = $r^3\omega^2$ hence $M = \frac{\text{gradient}}{G}$ or $r^3 = GM \times 1/\omega^2$ so gradient = GM hence $M = \frac{\text{gradient}}{G}$	A1
6(c)(ii)	$M = 4.1 \times 10^{23} / (6.0 \times 10^7 \times 6.67 \times 10^{-11})$ = $\mathbf{1.0 \times 10^{26}}$ kg	B1

Question	Answer	Marks
6(c)(iii)	$\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\frac{GM}{r} = v^2$	C1
	$v^2 = \frac{6.67 \times 10^{-11} \times 1.0 \times 10^{26}}{1.2 \times 10^8}$ $v^2 = 5.6 \times 10^7\ \text{m s}^{-1}$	C1
	$v = 7500\ \text{m s}^{-1}$	A1

Question	Answer	Marks
7(a)	work done per unit mass	**B1**
	(work done in) moving mass from infinity	**B1**
7(b)(i)	(gravitational) fields from the Earth and Moon are in opposite directions	**B1**
	(resultant is zero where gravitational) fields are equal (in magnitude)	**B1**
7(b)(ii)	$g \propto M / r^2$	**C1**
	$5.98 \times 10^{24} / x^2 = 7.35 \times 10^{22} / (3.84 \times 10^8 - x)^2$ leading to $x = 3.5 \times 10^8$ (m)	**A1**
7(b)(iii)	ϕ (Earth) = (–)$6.67 \times 10^{-11} \times (5.98 \times 10^{24} / 3.5 \times 10^8)$ **and** ϕ (Moon) = (–)$6.67 \times 10^{-11} \times (7.35 \times 10^{22} / 0.38 \times 10^8)$	**C1**
	ϕ = (–)$6.67 \times 10^{-11} \times [(5.98 \times 10^{24} / 3.5 \times 10^8) + (7.35 \times 10^{22} / 0.38 \times 10^8)]$	**C1**
	$= -1.3 \times 10^6$ J kg^{-1}	**A1**

Question	Answer	Marks
8(a)	force per unit mass	**B1**
8(b)	$GMm / r^2 = mr\omega^2$ **and** $\omega = 2\pi/T$ **or** $GMm / r^2 = mv^2 / r$ **and** $v = 2\pi r / T$	**C1**
	$6.67 \times 10^{-11} \times 6.0 \times 10^{24} = r^3 \times [2\pi / (94 \times 60)]^2$	**C1**
	$r = 6.9 \times 10^6$ m	**A1**
8(c)(i)	$r^3\omega^2$ = constant **or** r^3 / T^2 = constant	**C1**
	$r^3 / (6.9 \times 10^6)^3 = (150 / 94)^2$ so $r = 9.4 \times 10^6$ m	**A1**
	or	
	$GMT^2/4\pi^2 = r^3$ **and** clear that M is 6.0×10^{24}	**(C1)**
	$6.67 \times 10^{-11} \times 6.0 \times 10^{24} = r^3 \times [2\pi / (150 \times 60)]^2$ so $r = 9.4 \times 10^6$ m	**(A1)**
8(c)(ii)	separation increases so (potential energy) increases **or** movement is against gravitational force so (potential energy) increases	**B1**
8(c)(iii)	potential energy = (–)GMm / r	**C1**
	$\Delta E_P = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1200 \times [(6.9 \times 10^6)^{-1} - (9.4 \times 10^6)^{-1}]$	**C1**
	$= 1.9 \times 10^{10}$ J	**A1**

Question	Answer	Marks
9(a)	(gravitational) force per unit mass	**B1**
9(b)(i)	$g = GM / r^2$	**C1**
	$= (6.67 \times 10^{-11} \times 6.42 \times 10^{23}) / (3.39 \times 10^6)^2$ $= 3.73$ N kg^{-1}	**A1**
9(b)(ii)	$a = r\omega^2$ **and** $\omega = 2\pi / T$ **or** $a = v^2 / r$ **and** $v = 2\pi r / T$	**C1**
	$a = 3.39 \times 10^6 \times (2\pi / (24.6 \times 3600))^2$ $= 0.0171$ m s^{-2}	**A1**
9(b)(iii)	force per unit mass = 3.73 – 0.0171 = 3.71 N kg^{-1}	**A1**

Question	Answer
10(a)	(gravitational) force is (directly) proportional to product o masses
	force (between point masses) is inversely proportional to the square o thei separation
10(b)	correct read offs from the graph with correct power of ten for R^3
	$M = \frac{4\times\pi^2\times1.2\times10^{34}}{6.67\times10^{-11}\times2.4\times(365\times24\times3600)^2}$
	$=3.0\times10^{30}$ kg
10(c)(i)	potential energy is zero at infinity
	(gravitational) forces are attractive
	work must be done on the rock to move it to infinity
10(c)(ii)	$\frac{GMm}{r^2}=\frac{mv^2}{r}$ *OR* $v^2=\frac{GM}{r}$ *OR* $v=\sqrt{\frac{GM}{r}}$
	use of ½ mv^2 (e.g. multiplication by ½ m) leading to $\frac{GMm}{2r}$
10(c)(iii)	$Ep = \phi m$ and $\phi = \frac{-GM}{r}$ or $E_p = \frac{-GMm}{r}$ Total energy = $E_k + E$
	Total energy $= \frac{GMm}{2r} + \frac{-GMm}{r} = \frac{-GMm}{2r}$
11(a)	work done per unit mass
	(work done) moving mass from infinity (to the point)
11(b)(i)	gravitational potential energy = (–)GMm / r
	$\Delta E_P = 6.67\times10^{-11}\times6.0\times10^{24}\times2.4\times10^3\times[(6.4\times10^6)^{-1}-(1.2\times10^7)^{-1}]$
	or
	$\Delta\phi = 6.67\times10^{-11}\times6.0\times10^{24}\times[(6.4\times10^6)^{-1}-(1.2\times10^7)^{-1}]$
	$\Delta E_P = m\Delta\phi$
	$\Delta E_P = 7.0\times10^{10}$ J
11(b)(ii)	$GMm / r^2 = mv^2 / r$
	$v^2 = GM / r$ $= (6.67\times10^{-11}\times6.0\times10^{24}) / (1.2\times10^7)$
	$v = 5800$ m s^{-1}

11(c)	any one point from: • smaller gain in energy required if orbit is west to east • smaller change in velocity if orbit is west to east • smaller gain in energy if orbit is in same direction as Earth's rotation • smaller change in velocity if orbit is in same direction as Earth's rotation • satellite already moving west to east at launch • Earth's rotation is from west to east
12(a)(i)	region (of space)
	where a particle experiences a force
12(a)(ii)	force per unit mass
12(b)	$g = GM / R^2$
	$= (6.67 \times 10^{-11} \times 6.42 \times 10^{23}) / (3.39 \times 10^6)^2$
	$= 3.73$ N kg^{-1}
12(c)	$0.99 \times 3.73 = (6.67 \times 10^{-11} \times 6.42 \times 10^{23}) / r^2$
	$r = 3.41 \times 10^6$ (m)
	height $= (r - R)$ $= 2 \times 10^4$ m
	or
	$0.99 \times 3.73 = (6.67 \times 10^{-11} \times 6.42 \times 10^{23}) / (R + h)^2$ $(R + h)^2 = 1.1596 \times 10^{13}$
	$R + h = 3.41 \times 10^6$ (m)
	$h = 2 \times 10^4$ m
13(a)	work done per unit mass
	(work done to) move mass from infinity (to the point)
13(b)	curve from r to $4r$, with gradient of decreasing magnitude and starting at $(r, \pm\phi)$
	line passing through $(2r, \pm 0.5\phi)$ and $(4r, \pm 0.25\phi)$
	line showing potential is negative throughout
13(c)(i)	gravitational potential energy = (–) GMm / R
	change $= (6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 3.4 \times 10^3) / (6.4 \times 10^6) \times [1/3 - 1/4]$
	$= 1.8 \times 10^{10}$ J
13(c)(ii)	rock loses potential energy
	(so) kinetic energy increases so speed increases
	or
	force is attractive
	moves towards planet so speeds up

14(a)	work done per unit mass
	work done moving mass from infinity (to the point)
14(b)(i)	gravitational force provides centripetal force
	$mv^2 / r = GMm / r^2$ and $v = 2\pi r / T$ OR $mr\omega^2 = GMm / r^2$ and $\omega = 2\pi / T$ OR $r^3 = GMT^2 / 4\pi^2$
	$r^3 = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (13.7 \times 24 \times 3600)^2 / 4\pi^2$ so $r = 2.4 \times 10^8$ m
14(b)(ii)	($E_P = -$) GMm / r work done = $GMm / r_1 - GMm / r_2$
	$= 6.67 \times 10^{-11} \times 360 \times 6.0 \times 10^{24}$ $(1/6.4 \times 10^6 - 1 / 2.4 \times 10^8)$
	= **2.2×10^{10}** J
14(b)(iii)	$g = GM / r^2$
	ratio $= r_{TESS}^2 / r_{earth}^2$ $= (2.4 \times 10^8 / 6.4 \times 10^6)^2$ = **1400**

15(a)	force proportional to product of masses and inversely proportional to square of separation
	idea of (gravitational) force between point masses
15(b)(i)	above the equator
	from west to east
15(b)(ii)	gravitational force provides/is the centripetal force
	$GM / r^2 = r(2\pi / T)^2$
	$M = 6.0 \times 10^{24}$ kg

16(a)	(F =) GMm / x^2, where G is the (universal) gravitational constant
16(b)(i)	angle = $(1.2 \times 10^{-3}) / (8.0 \times 10^{-2}) = 1.5 \times 10^{-2}$ (rad)
16(b)(ii)	torque $= 1.5 \times 10^{-2} \times 9.3 \times 10^{-10}$ $= 1.4 \times 10^{-11}$ N m
16(c)(i)	force $\times 8.0 \times 10^{-2} = 1.4 \times 10^{-11}$
	$(G \times 1.3 \times 7.5 \times 10^{-3} \times 8.0 \times 10^{-2}) / (6.0 \times 10^{-2})^2 = 1.4 \times 10^{-11}$
	$G = 6.4 \times 10^{-11}$ N m^2 kg^{-2}
16(c)(ii)	Any one from: • law applies only to point masses/spheres are not point masses • radii of spheres not small compared with separation • spheres may not be uniform • the masses are not isolated • force between L and rod • spheres may be charged/may be electrostatic force (between spheres)

17(a)	(F =) GMm / x^2, where G is the (universal) gravitational constant
17(b)(i)	$GMm / x^2 = mv^2 / x$ **or** $v^2 = GM / x$
	$v^2 = (6.67 \times 10^{-11} \times 7.5 \times 10^{23}) / (3.4 \times 10^6 + 240 \times 10^3)$ so $v = 3.7 \times 10^3$ m s^{-1}
17(b)(ii)	potential energy = (–)GMm / x
	$E_A = (–)(6.67 \times 10^{-11} \times 7.5 \times 10^{23} \times 650) / (3.64 \times 10^6)$ **or** $E_B = (–)(6.67 \times 10^{-11} \times 7.5 \times 10^{23} \times 650) / (5.00 \times 10^7)$
	correct substitution and subtraction $E_B – E$ shown, leading to $\Delta E_p = 8.3 \times 10^9$ J
	or
	ϕ = (–)GM / x **and** potential energy = $m\phi$
	$\Delta\phi = (6.67 \times 10^{-11} \times 7.5 \times 10^{23}) \times [(1 / (3.64 \times 10^6)) – (1 / (5.00 \times 10^7))]$ (= 1.27×10^7 J kg^{-1})
	$\Delta E_p = 1.27 \times 10^7 \times 650$ $= 8.3 \times 10^9$ J
17(c)	kinetic energy <u>or</u> potential energy decreases
	kinetic energy <u>and</u> potential energy decrease so total energy decreases

18(a)(i)	work done per unit mass
	idea of work done moving mass from infinity (to the point)
18(a)(i)	(gravitational) force is attractive
	(gravitational) potential at infinity is zero
	decrease in potential energy as masses approach *or* displacement and force in opposite directions
18(b)(i)	*Either* $mv^2 / R = GMm / R^2$ *Or* $v = \sqrt{(GM / R)}$ $v^2 = (6.67 \times 10^{-11} \times 6.00 \times 10^{24}) / (7.30 \times 10^6)$
	giving $v = 7.4 \times 10^3$ m s^{-1}
18(b)(ii)	$V_P = – GMm / R$
	$= – (6.67 \times 10^{-11} \times 6.00 \times 10^{24} \times 340) / (7.30 \times 10^6)$
	$V_P = – 1.9 \times 10^{10}$ J
18(c)	$v^2 \propto 1 / r$, (r smaller) so v greater
	and E_K greater

19(a)(i)	force per unit mass
19(a)(ii)	acceleration= F/m, field strength = F/m, so equal
19(b)	smooth curve between R and $4R$ with negative gradient of decreasing magnitude
	line passing through (R, 1.00g) and (2R, 0.25g)
	line ending at (4R, 0.0625g)
19(c)	$M = (4/3 \times \pi R^3)\rho$
	$g = GM/(2R)^2$
	$g = ⅓ \times 6.67 \times 10^{11} \times \pi \times 3.4 \times 10^6 \times 4.0 \times 10^3$ $= 0.95\,\text{m}\,\text{s}^{-2}$

Q20

(i)	direction of the force acting on a (test) mass placed at the point	B1
1(a)(ii)	change in height negligible compared with radius (of Earth)	B1
	(so) field lines are (effectively) parallel	B1
1(b)(i)	$Y = GM/R^2$	M1
	G is the gravitational constant	A1
1(b)(ii)	gravitational force is (always) attractive **or** gravitational force (always) acts towards the centre of the sphere	B1
	force is in opposite direction to displacement **or** at a point to the right of the centre, force acts to the left **or** at a point to the left of the centre, force acts to the right	B1
1(b)(iii)	sketch: smooth curve with decreasing positive gradient, starting at (R, –Y) and reaching 3R with g still negative **or** smooth curve with increasing positive gradient, ending at (–R, Y) and reaching –3R with g still positive	B1
	both of the above curves, in correct quadrants	B1
	curve passing through (±2R, ±0.25Y) and (±3R, ±0.11Y)	B1

Q21

(i)	work done per unit mass	B1
	work (done) moving mass from infinity (to the point)	B1
2(a)(ii)	$\phi = -GM/r$ $= -(6.67 \times 10^{-11} \times 7.3 \times 10^{22})/(1.7 \times 10^6)$	C1
	$= -2.9 \times 10^6\,\text{J}\,\text{kg}^{-1}$	A1
2(b)(i)	$E_P = m\phi$	B1
2(b)(ii)	$½mv^2 + m\phi = 0$	M1
	correct algebra leading to $v = \sqrt{(-2\phi)}$	A1
2(c)	speed = $\sqrt{(2 \times 2.9 \times 10^6)}$ $= 2400\,\text{m}\,\text{s}^{-1}$	A1

Q22 (a)	(gravitational) potential is zero at infinity (gravitational force between two masses is attractive so)	B1
	either work is done on a mass to move it away from another mass **or** work is done on a mass to move it to infinity	B1
1(b)(i)	M = (–) gradient / G	C1
	e.g. $M = (1.76 \times 10^{8}) / (3.0 \times 10^{-8} \times 6.67 \times 10^{-11}) = 8.8 \times 10^{25}$ kg	A1
1(b)(ii)	**either** $GMm / r^2 = mr\omega^2$ **and** $\omega = 2\pi / T$ **or** $GMm / r^2 = mv^2 / r$ **and** $v = 2\pi r / T$ **or** $GMm / r^2 = 4\pi^2 mr / T^2$	C1
	$R^3 = 6.67 \times 10^{-11} \times 8.8 \times 10^{25} \times (0.72 \times 24 \times 60 \times 60)^2 / 4\pi^2$	C1
	$R = 8.3 \times 10^{7}$ m	A1
1(b)(iii)	$\Delta E = (GMm / r) - ½mv^2$ kinetic energy = $(½ \times 1200 \times 8400^2)$	C1
	potential energy = $(-)[(6.67 \times 10^{-11} \times 8.8 \times 10^{25} \times 1200) / (8.3 \times 10^{7})]$	C1
	$\Delta E = [(6.67 \times 10^{-11} \times 8.8 \times 10^{25} \times 1200) / (8.3 \times 10^{7})] - (½ \times 1200 \times 8400^2)$ $= 4.3 \times 10^{10}$ J	A1

Q23 (a)	work done per unit mass	B1
	work done moving mass from infinity (to the point)	B1
1(b)(i)	potential is zero at infinity	B1
	work is done **by** (two) masses in moving them **closer** together **or** work is done **on** (two) masses in moving them **apart**	B1
1(b)(ii)	magnitude of potential shown as 4Φ	B1
	potential negative **and** shown as a multiple of $-\Phi$ [potential = -4Φ if fully correct]	B1
1(b)(iii)	field strength at X: $\Phi / 4R$	A1
	field strength at Y: $4\Phi / R$	A1
	potential energy at X: $-M\Phi$	A1
	potential energy at Y: $-8M\Phi$	A1