SIMPLE HARMONIC MOTION
A Level Physics 9702

MJ23/42/Q4

1 A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

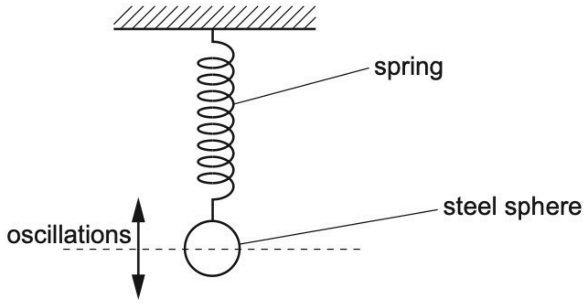

Fig. 4.1

The velocity v of the sphere varies with displacement x from its equilibrium position according to

$$v = \pm\, 9.7\sqrt{(11.6 - x^2)}$$

where v is in $\text{cm}\,\text{s}^{-1}$ and x is in cm.

(a) **(i)** Calculate the frequency of the oscillations.

frequency = ... Hz [2]

(ii) Show that the amplitude of the oscillations is 3.4 cm.

[1]

(iii) Calculate the maximum acceleration a_0 of the sphere.

a_0 = ... $\text{m}\,\text{s}^{-2}$ [2]

(b) On Fig. 4.2, sketch the variation with x of the acceleration a of the sphere.

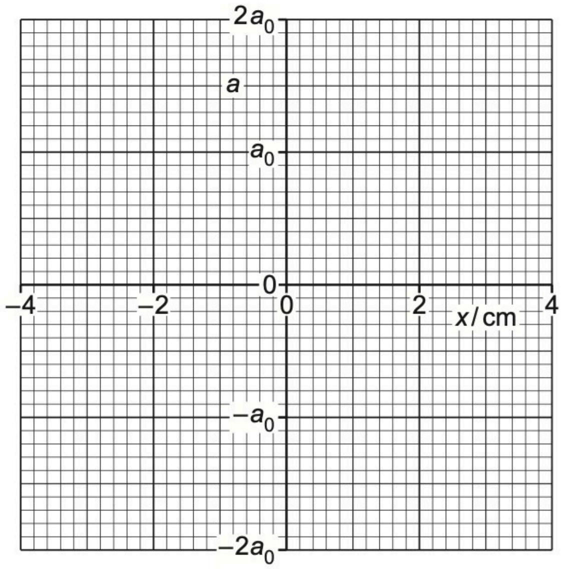

Fig. 4.2

[3]

(c) Describe, without calculation, the interchange between the potential energy and the kinetic energy of the oscillations.

....

....

....

....

.... [3]

[Total: 11]

2 An object is suspended from a vertical spring as shown in Fig. 3.1. **FM23/42/Q3**

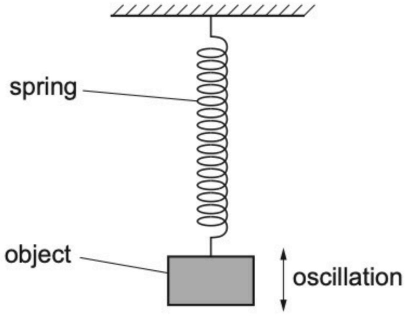

Fig. 3.1

The object is displaced vertically and then released so that it oscillates, undergoing simple harmonic motion.

Fig. 3.2 shows the variation with displacement x of the energy E of the oscillations.

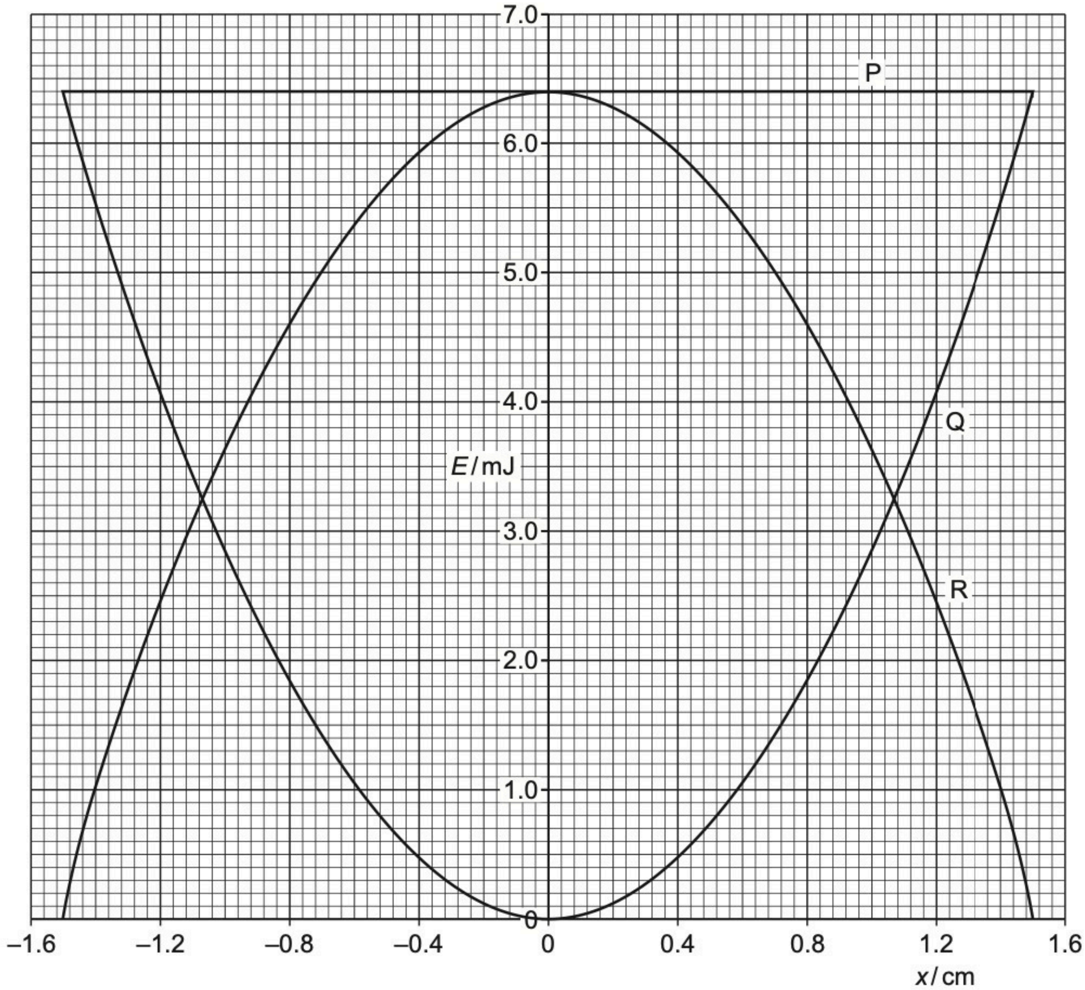

Fig. 3.2

The kinetic energy, the potential energy and the total energy of the oscillations are each represented by one of the lines P, Q and R.

(a) State the energy that is represented by each of the lines P, Q and R.

P ....

Q ....

R ....
[2]

(b) The object has a mass of 130 g.
Determine the period of the oscillations.

period = ... s [4]

(c) **(i)** State the cause of damping.

....

.... [1]

(ii) A light card is attached to the object. The object is displaced with the same initial amplitude and then released. During each complete oscillation the total energy of the system decreases by 8.0% of the total energy at the start of that oscillation.

Determine the decrease in total energy, in mJ, of the system by the end of the first 6 complete oscillations.

energy lost = ... mJ [2]

(iii) State, with a reason, the type of damping that the card introduces into the system.

....

....

.... [1]

[Total: 10]

3 Fig. 4.1 shows the variation with time t of the height h above the ground of an object of mass 36 kg that is undergoing vertical simple harmonic motion. **ON22/42/Q4**

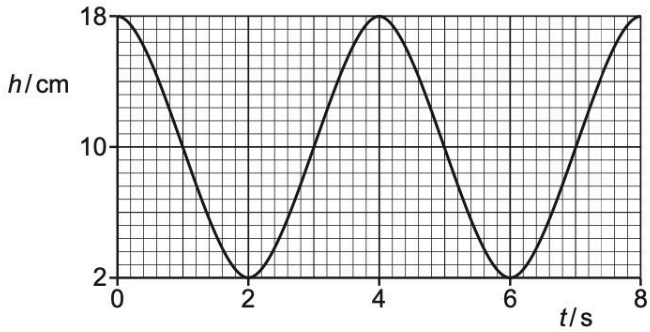

Fig. 4.1

(a) For the oscillations of the object:

(i) determine the amplitude x_0, in cm

x_0 = ... cm [1]

(ii) show that the angular frequency ω is 1.6 rad s^{-1}

[2]

(iii) determine the total energy E.

E = ... J [3]

(b) On Fig. 4.2, sketch the variation with h of the kinetic energy E_K of the object.

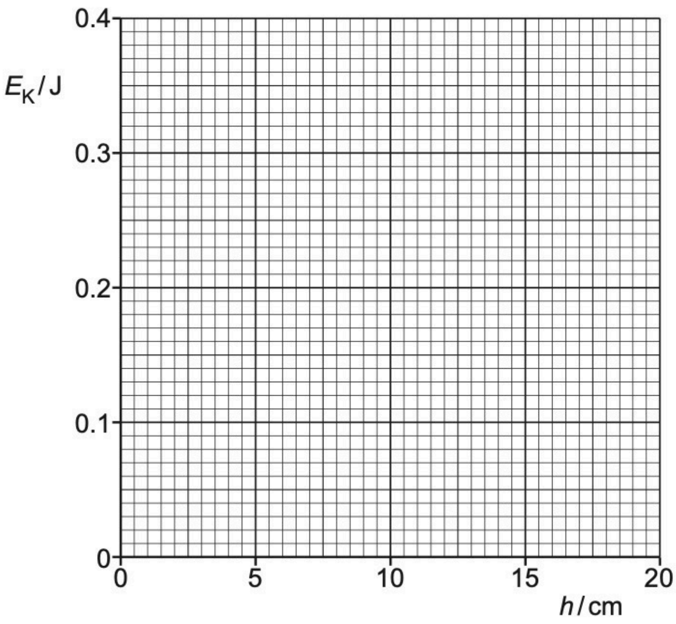

Fig. 4.2

[4]

[Total: 10]

4 An object is suspended from a spring that is attached to a fixed point as shown in Fig. 3.1.

ON22/41/Q3

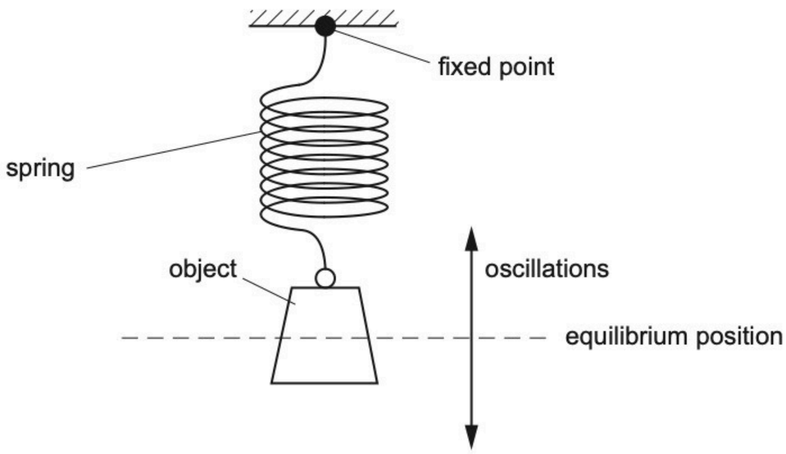

Fig. 3.1

The object oscillates vertically with simple harmonic motion about its equilibrium position.

(a) State the defining equation for simple harmonic motion. Identify the meaning of each of the symbols used to represent physical quantities.

....

....

.... [2]

(b) The variation with displacement x from the equilibrium position of the velocity v of the object is shown in Fig. 3.2.

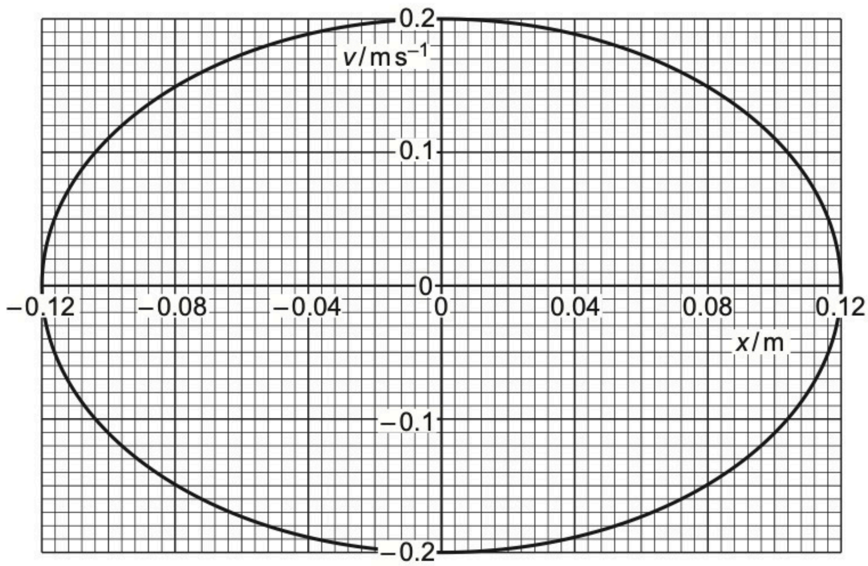

Fig. 3.2

The variation with x of the potential energy E_P of the oscillations of the object is shown in Fig. 3.3.

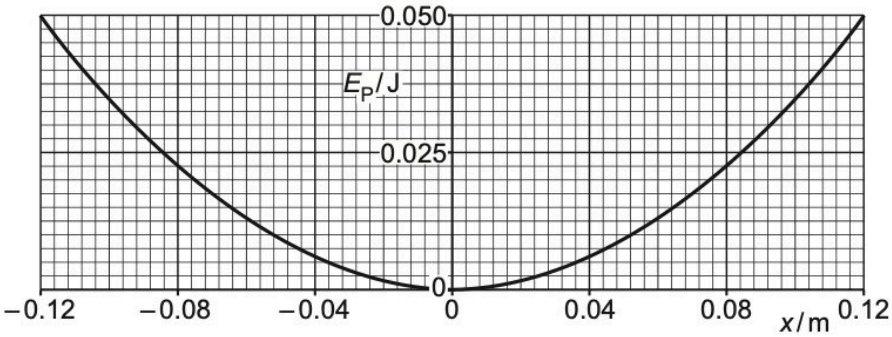

Fig. 3.3

Use Fig. 3.2 and Fig. 3.3 to:

(i) determine the amplitude x_0 of the oscillations

x_0 = ... m [1]

(ii) show that the angular frequency of the oscillations is 1.7 rad s^{-1}

[2]

(iii) determine the mass M of the object.

M = ... kg [2]

(c) The oscillations of the object are now lightly damped.

(i) State what is meant by damping.

....

....

.... [2]

(ii) Assume that the damping does not change the angular frequency of the oscillations.

On Fig. 3.2, sketch the variation with x of v when the amplitude of the oscillations is 0.060 m. [2]

[Total: 11]

5 **(a)** State what is meant by resonance. **MJ22/42/Q4**

....

....

.... [2]

(b) Fig. 4.1 shows a heavy pendulum and a light pendulum, both suspended from the same piece of string. This string is secured at each end to fixed points.

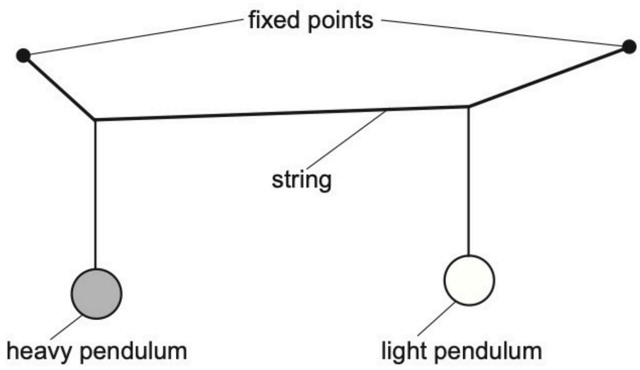

Fig. 4.1

Both pendulums have the same natural frequency.
The heavy pendulum is set oscillating perpendicular to the plane of the diagram. As it oscillates, it causes the light pendulum to oscillate.

Fig. 4.2 shows the variation with time t of the displacements of the two pendulums for three oscillations.

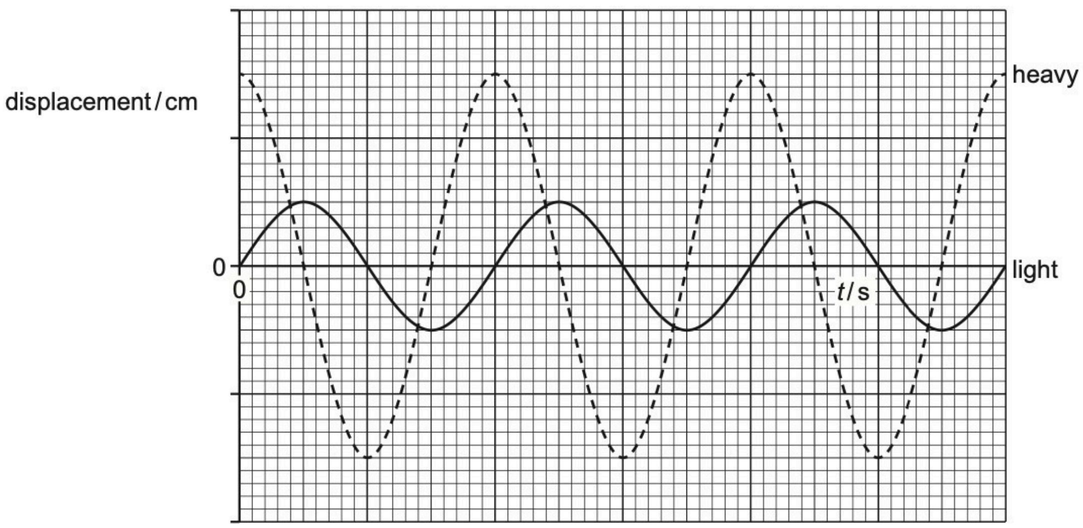

Fig. 4.2

The variation with t of the displacement x of the light pendulum is given by

$$x = 0.25 \sin 5.0\pi t$$

where x is in centimetres and t is in seconds.

(i) Calculate the period T of the oscillations.

T = ... s [2]

(ii) On Fig. 4.2, label both of the axes with the correct scales. Use the space below for any additional working that you need.

[2]

(iii) Determine the magnitude of the phase difference ϕ between the oscillations of the light and heavy pendulums. Give a unit with your answer.

ϕ = ... unit .... [2]

[Total: 8]

6 A pendulum consists of a bob (small metal sphere) attached to the end of a piece of string. The other end of the string is attached to a fixed point. The bob oscillates with small oscillations about its equilibrium position, as shown in Fig. 4.1.

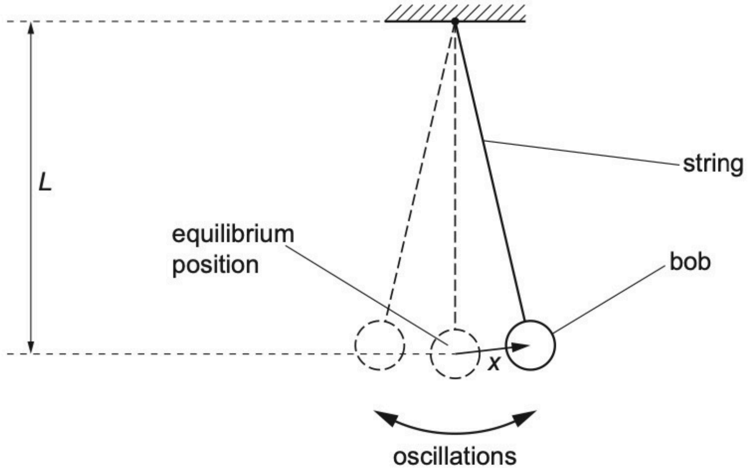

Fig. 4.1 (not to scale)

The length L of the pendulum, measured from the fixed point to the centre of the bob, is 1.24 m.

The acceleration a of the bob varies with its displacement x from the equilibrium position as shown in Fig. 4.2.

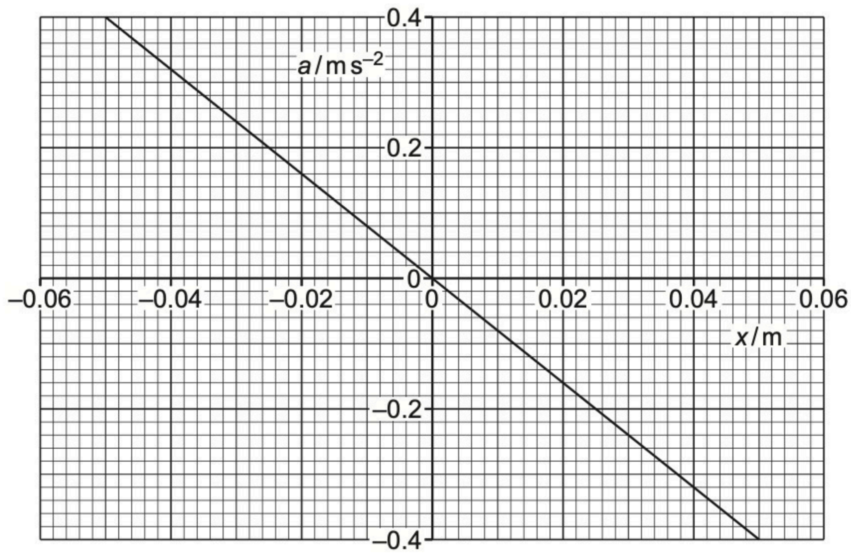

Fig. 4.2

(a) State how Fig. 4.2 shows that the motion of the pendulum is simple harmonic.

....

....

.... [2]

(b) **(i)** Use Fig. 4.2 to determine the angular frequency ω of the oscillations.

ω = ... rad s^{-1} [2]

(ii) The angular frequency ω is related to the length L of the pendulum by

$$\omega = \sqrt{\frac{k}{L}}$$

where k is a constant.

Use your answer in **(b)(i)** to determine k. Give a unit with your answer.

k = ... unit .... [2]

(c) While the pendulum is oscillating, the length of the string is increased in such a way that the total energy of the oscillations remains constant.

Suggest and explain the qualitative effect of this change on the amplitude of the oscillations.

....

....

.... [2]

[Total: 8]

7 A small wooden block (cuboid) of mass m floats in water, as shown in Fig. 3.1. **FM22/42/Q3**

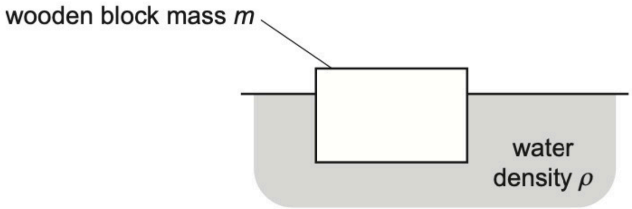

Fig. 3.1

The top face of the block is horizontal and has area A. The density of the water is ρ.

(a) State the names of the two forces acting on the block when it is stationary.

.... [1]

(b) The block is now displaced downwards as shown in Fig. 3.2 so that the surface of the water is higher up the block.

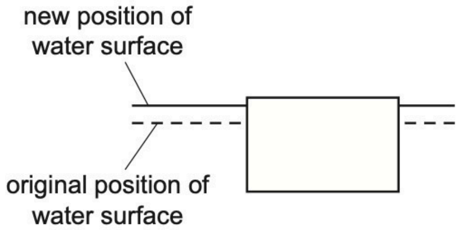

Fig. 3.2

State and explain the direction of the resultant force acting on the wooden block in this position.

....

.... [1]

(c) The block in **(b)** is now released so that it oscillates vertically.

The resultant force F acting on the block is given by

$$F = -Ag\rho x$$

where g is the gravitational field strength and x is the vertical displacement of the block from the equilibrium position.

(i) Explain why the oscillations of the block are simple harmonic.

....

....

.... [2]

(ii) Show that the angular frequency ω of the oscillations is given by

$$\omega = \sqrt{\frac{A\rho g}{m}}.$$

[2]

(d) The block is now placed in a liquid with a greater density. The block is displaced and released so that it oscillates vertically. The variation with displacement x of the acceleration a of the block is measured for the first half oscillation, as shown in Fig. 3.3.

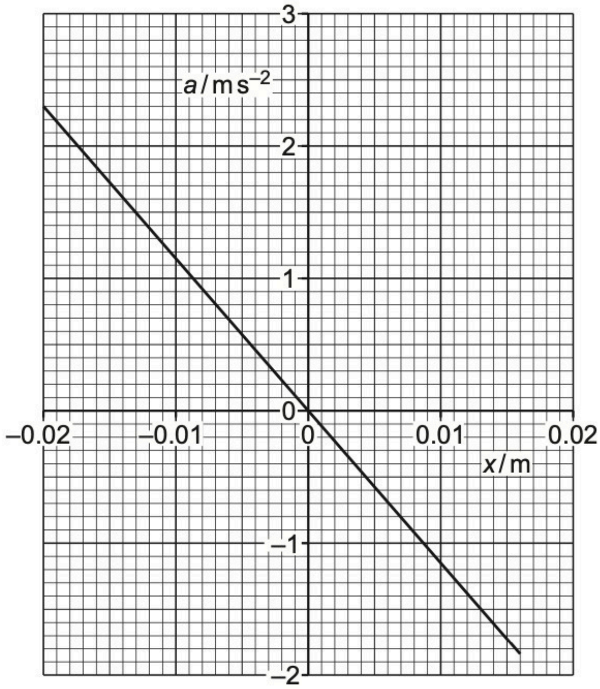

Fig. 3.3

(i) Explain why the maximum negative displacement of the block is not equal to its maximum positive displacement.

....

....

.... [1]

(ii) The mass of the block is 0.57 kg.

Use Fig. 3.3 to determine the decrease ΔE in energy of the oscillation for the first half oscillation.

E = ... J [3]

[Total: 10]

8 **(a)** The defining equation of simple harmonic motion is *F/M/21/42/Q4*

$$a = -\omega^2 x.$$

State the significance of the minus (–) sign in the equation.

....

.... [1]

(b) A trolley rests on a bench. Two identical stretched springs are attached to the trolley as shown in Fig. 4.1. The other end of each spring is attached to a fixed support.

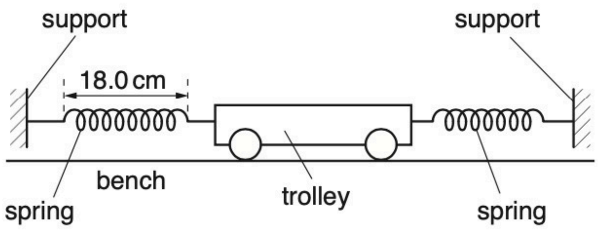

Fig. 4.1

The unstretched length of each spring is 12.0 cm. The spring constant of each spring is $8.0\,\mathrm{N\,m^{-1}}$. When the trolley is in equilibrium the length of each spring is 18.0 cm.

The trolley is displaced 4.8 cm to one side and then released. Assume that resistive forces on the trolley are negligible.

(i) Show that the resultant force on the trolley at the moment of release is 0.77 N.

'rus Ishaq

[2]

(ii) The mass of the trolley is 250 g.

Calculate the maximum acceleration *a* of the trolley.

a = ... $m s^{-2}$ [1]

(iii) Use your answer in (ii) to determine the period *T* of the subsequent oscillation.

T = ... s [3]

(iv) The experiment is repeated with an initial displacement of the trolley of 2.4 cm.

State and explain the effect, if any, this change has on the period of the oscillation of the trolley.

....

....

.... [2]

[Total: 9]

9 A U-shaped tube contains some liquid. The liquid column in each half of the tube has length L, as shown in Fig. 3.1.

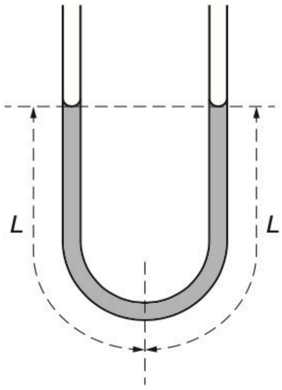

Fig. 3.1

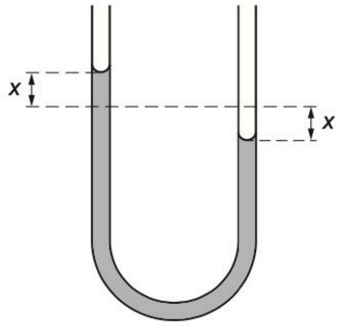

Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration a of the liquid in the tube is related to the displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

where g is the acceleration of free fall.

(a) Explain how the expression shows that the liquid in the tube is undergoing simple harmonic motion.

....

....

....

....

.... [3]

(b) The length L of each liquid column is 18 cm.

Determine the period T of the oscillations.

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, ISL, LGS, Learning Alliance)

T = ... s [3]

(c) The oscillations of the liquid in the tube are damped.
In any one complete cycle of the oscillations, the amplitude decreases by 6.0% of its value at the beginning of the oscillation.

Determine the ratio

$$\frac{\text{energy of oscillations after 3 cycles}}{\text{initial energy of oscillations}}.$$

ratio = ... [3]

[Total: 9]

10 (a) State what is meant by *simple harmonic motion*. *M/J/21/43/Q3*

....

....

.... [2]

(b) A trolley of mass m is held on a horizontal surface by means of two springs. One spring is attached to a fixed point P. The other spring is connected to an oscillator, as shown in Fig. 3.1.

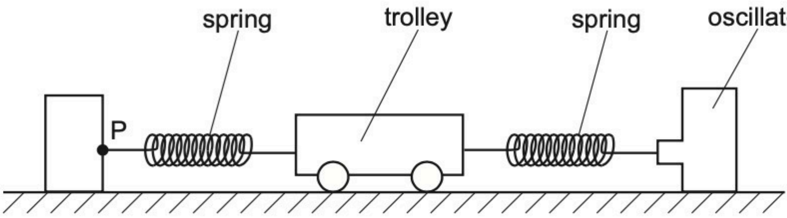


Fig. 3.1

The springs, each having spring constant k of 130 N m^{-1}, are always extended.

The oscillator is switched off. The trolley is displaced along the line of the springs and then released. The resulting oscillations of the trolley are simple harmonic.

The acceleration a of the trolley is given by the expression

$$a = -\left(\frac{2k}{m}\right)x$$

where x is the displacement of the trolley from its equilibrium position.

The mass of the trolley is 840 g.

Calculate the frequency f of oscillation of the trolley.

f = ... Hz [3]

(c) The oscillator in **(b)** is switched on. The frequency of oscillation of the oscillator is varied, keeping its amplitude of oscillation constant.

The amplitude of oscillation of the trolley is seen to vary. The amplitude is a maximum at the frequency calculated in **(b)**.

(i) State the name of the effect giving rise to this maximum.

.... [1]

(ii) At any given frequency, the amplitude of oscillation of the trolley is constant.

Explain how this indicates that there are resistive forces opposing the motion of the trolley.

....

....

.... [2]

[Total: 8]

11 A pendulum consists of a metal sphere P suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

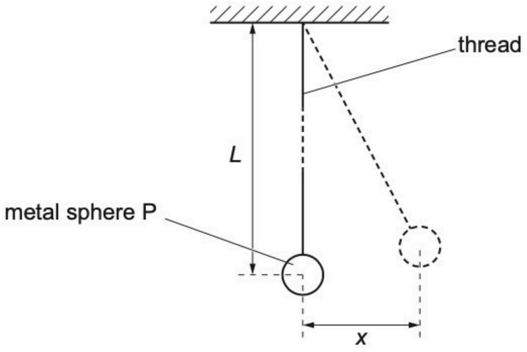


Fig. 3.1

The centre of gravity of sphere P is a distance *L* from the fixed point.

The sphere is pulled to one side and then released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

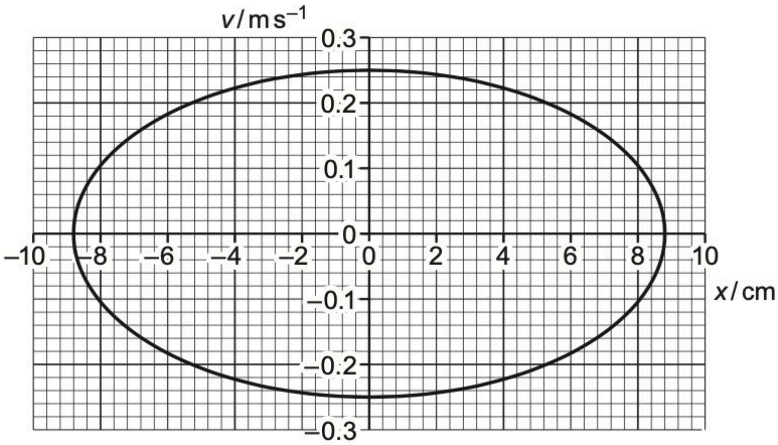
(a) State what is meant by *simple harmonic motion*.

....

....

.... [2]

(b) The variation of the velocity *v* of sphere P with the displacement *x* from its mean position is shown in Fig. 3.2.



Use Fig. 3.2 to determine the frequency *f* of the oscillations of sphere P.

f = ... Hz [3]

(c) The period *T* of the oscillations of sphere P is given by the expression

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

where *g* is the acceleration of free fall.

Use your answer in **(b)** to determine the length *L*.

L = ... m [2]

(d) Another pendulum consists of a sphere Q suspended by a thread. Spheres P and Q are identical. The thread attached to sphere Q is longer than the thread attached to sphere P.

Sphere Q is displaced and then released. The oscillations of sphere Q have the same amplitude as the oscillations of sphere P.

On Fig. 3.2, sketch the variation of the velocity *v* with displacement *x* for sphere Q. [2]

[Total: 9]

12 A mass is suspended vertically from a fixed point by means of a spring, as illustrated in Fig. 4.1.

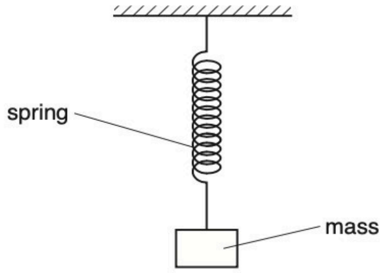


Fig. 4.1

The mass is oscillating vertically. The variation with displacement x of the acceleration a of the mass is shown in Fig. 4.2.

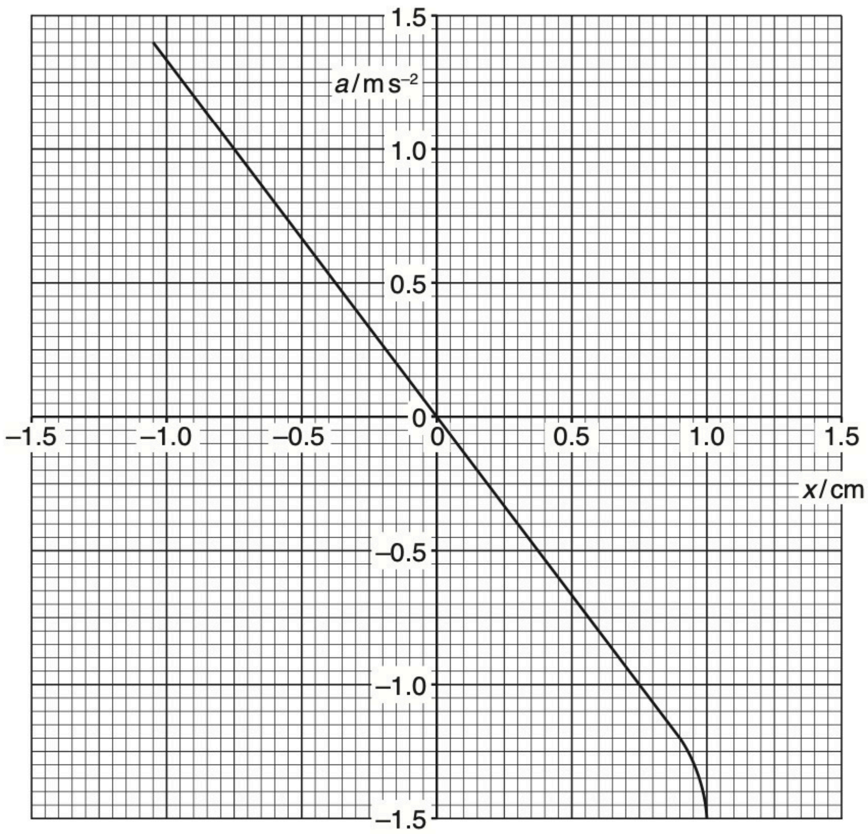


Fig. 4.2

(a) **(i)** State what is meant by the *displacement* of the mass on the spring.

....

....

(ii) Suggest how Fig. 4.2 shows that the mass is not performing simple harmonic motion.

....

.... [1]

(b) **(i)** The amplitude of oscillation of the mass may be changed.

State the maximum amplitude x_0 for which the oscillations are simple harmonic.

x_0 = ... cm [1]

(ii) For the simple harmonic oscillations of the mass, use Fig. 4.2 to determine the frequency of the oscillations.

frequency = ... Hz [3]

(c) The maximum speed of the mass when oscillating with simple harmonic motion of amplitude x_0 is v_0.

On Fig. 4.3, show the variation with displacement x of the velocity v of the mass for displacements from $+x_0$ to $-x_0$.

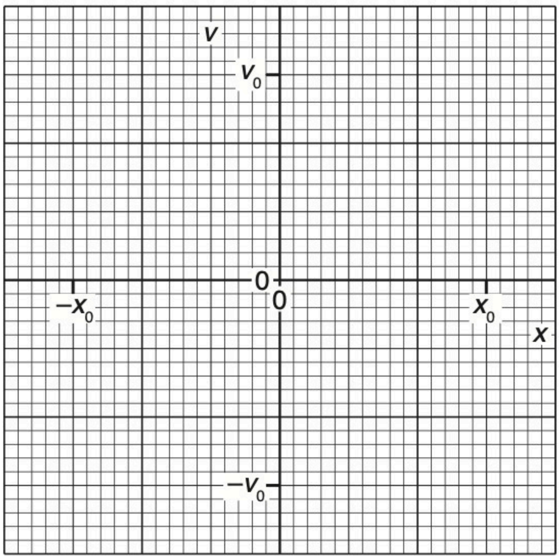


Fig. 4.3

aq

[2]

13 A simple pendulum consists of a metal sphere suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

O/N/20/42/Q3

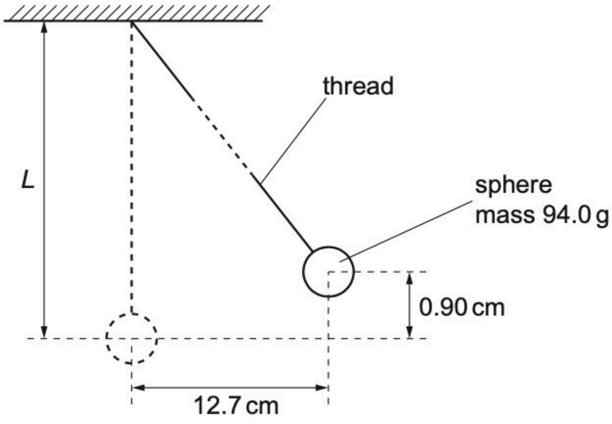


Fig. 3.1 (not to scale)

The sphere of mass 94.0 g is displaced to one side through a horizontal distance of 12.7 cm. The centre of gravity of the sphere rises vertically by 0.90 cm.

The sphere is released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

(a) State what is meant by *simple harmonic motion*.

....

....

.... [2]

(b) **(i)** State the kinetic energy of the sphere when the sphere returns to the displaced position shown in Fig. 3.1.

kinetic energy = ... J [1]

(ii) Calculate the total energy E_T of the oscillations.

(iii) Use your answer in **(ii)** to show that the angular frequency ω of the oscillations of the pendulum is $3.3\,rad\,s^{-1}$.

[2]

(c) The period T of oscillation of the pendulum is given by the expression

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

where g is the acceleration of free fall and L is the length of the pendulum.

Use data from **(b)** to determine L.

L = ... m [3]

[Total: 10]

14 **(a)** State two conditions necessary for a mass to be undergoing simple harmonic motion.

1. ....

....

2. ....

.... [2]

(b) A trolley of mass 950 g is held on a horizontal surface by means of two springs attached to fixed points P and Q, as shown in Fig. 4.1.

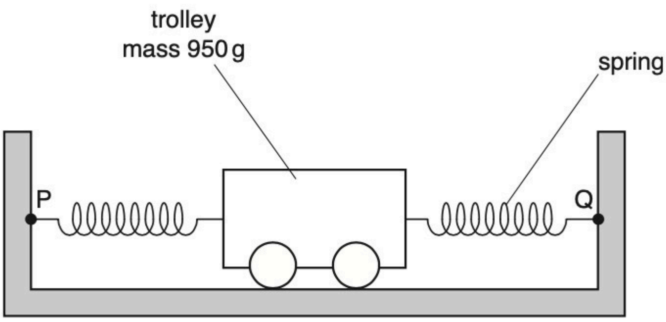
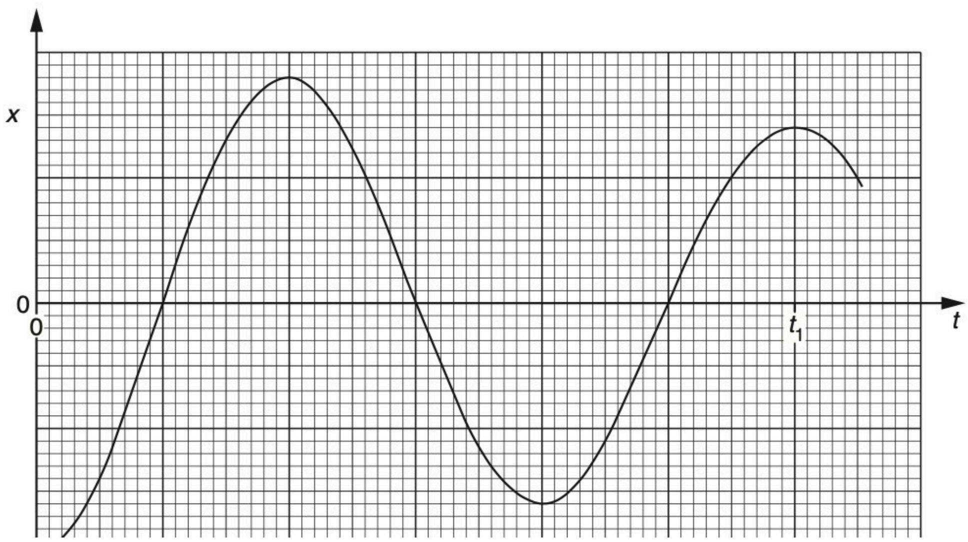


Fig. 4.1

The springs, each having a spring constant k of 230 N m^{-1}, are always extended.

The trolley is displaced along the line of the springs and then released.
The variation with time t of the displacement x of the trolley is shown in Fig. 4.2.



(i) **1.** State and explain whether the oscillations of the trolley are heavily damped, critically damped or lightly damped.

....

....

2. Suggest the cause of the damping.

....

....

....

[3]

(ii) The acceleration a of the trolley of mass m may be assumed to be given by the expression

$$a = -\left(\frac{2k}{m}\right)x.$$

1. Calculate the angular frequency ω of the oscillations of the trolley.

ω = ... rad s^{-1} [3]

2. Determine the time t_1 shown on Fig. 4.2.

t_1 = ... s [2]

[Total: 10]

15 A metal plate is made to vibrate vertically by means of an oscillator, as shown in Fig. 2.1.

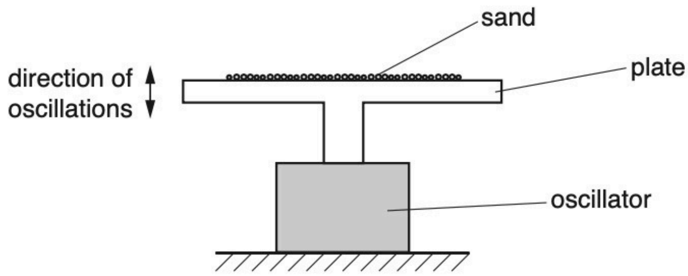


Fig. 2.1

Some sand is sprinkled on to the plate.

The variation with displacement y of the acceleration a of the sand on the plate is shown in Fig. 2.2.

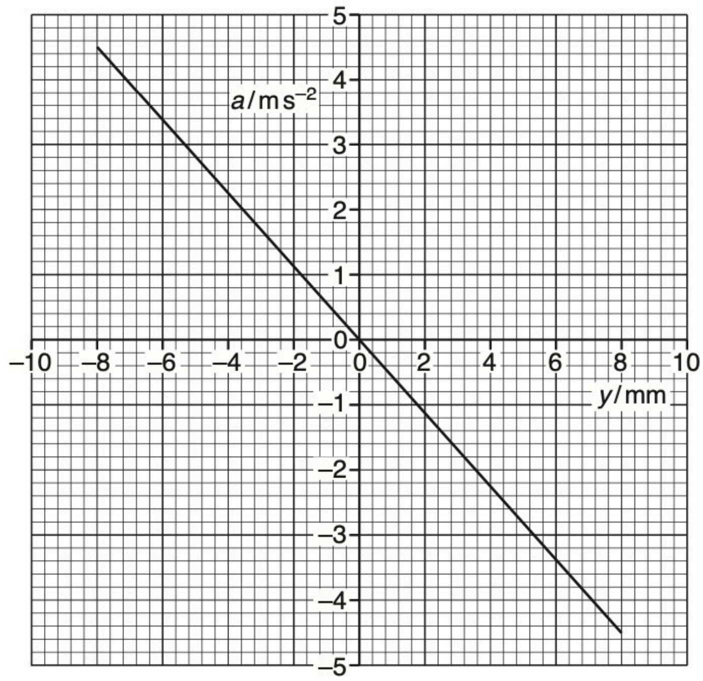


Fig. 2.2

(a) (i) Use Fig. 2.2 to show how it can be deduced that the sand is undergoing simple harmonic motion.

....

....

....

.... [2]

(ii) Calculate the frequency of oscillation of the sand.

frequency = ... Hz [2]

(b) The amplitude of oscillation of the plate is gradually increased beyond 8 mm. The frequency is constant.

At one amplitude, the sand is seen to lose contact with the plate.

For the plate when the sand first loses contact with the plate,

(i) state the position of the plate,

....[1]

(ii) calculate the amplitude of oscillation.

amplitude = ... mm [3]

[Total: 8]

16 **(a)** State, by reference to displacement, what is meant by *simple harmonic motion*.

....

....

....

.... [2]

(b) A mass is undergoing oscillations in a vertical plane.

The variation with displacement x of the acceleration a of the mass is shown in Fig. 3.1.

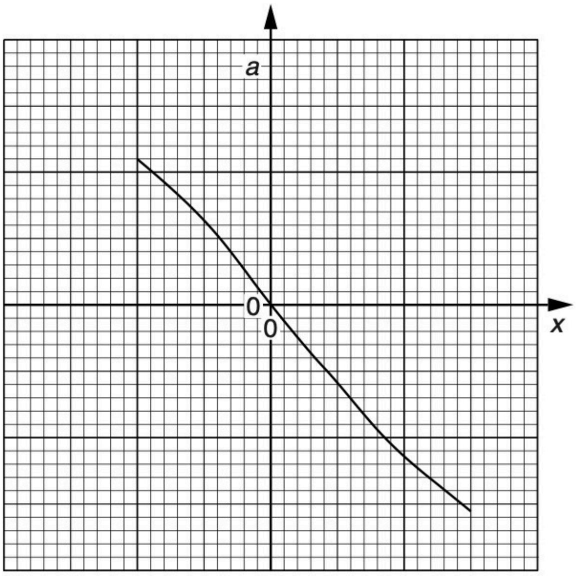


Fig. 3.1

State two reasons why the motion of the mass is not simple harmonic.

1. ....

....

2. ....

....
[2]

(c) A block of wood is floating in a liquid, as shown in Fig. 3.2.

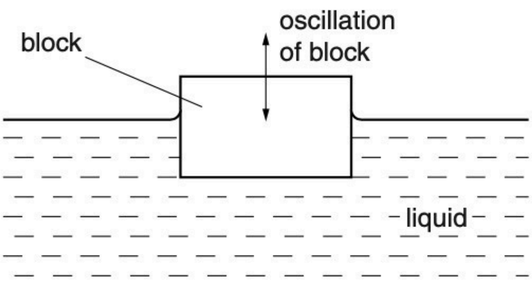


Fig. 3.2

The block is displaced vertically and then released.

The variation with time t of the displacement y of the block from its equilibrium position is shown in Fig. 3.3.

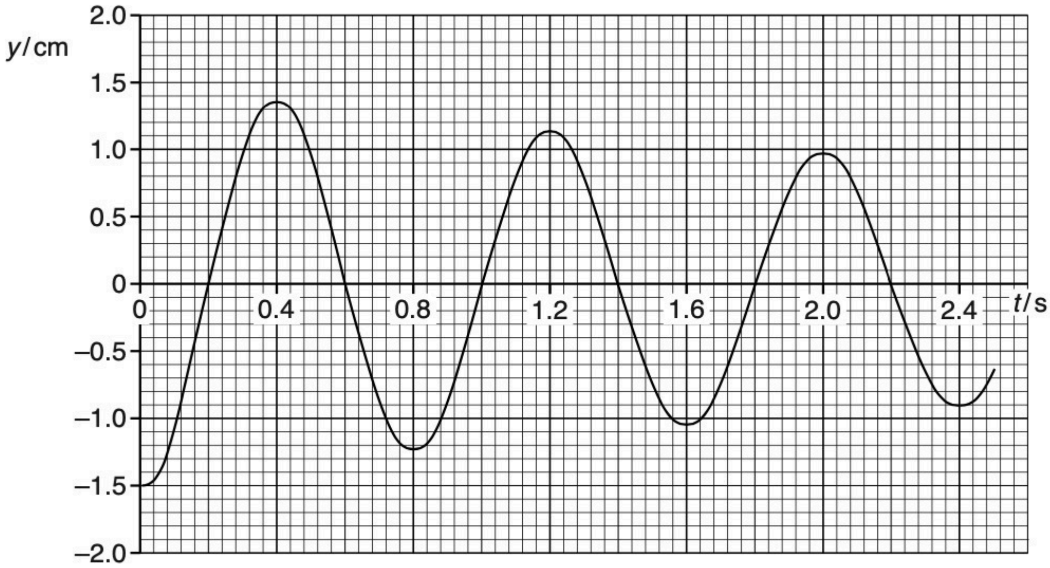


Fig. 3.3

Use data from Fig. 3.3 to determine

(i) the angular frequency ω of the oscillations,

LGS, Learning Alliance)

ω = ... rad s^{-1} [2]

(ii) the maximum vertical acceleration of the block.

maximum acceleration = ... $m s^{-2}$ [2]

(iii) The block has mass 120 g.

The oscillations of the block are damped. Calculate the loss in energy of the oscillations of the block during the first three complete periods of its oscillations.

energy loss = ... J [3]

[Total: 11]

17 A U-tube contains liquid, as shown in Fig. 3.1. *O/N/18/43/Q3*

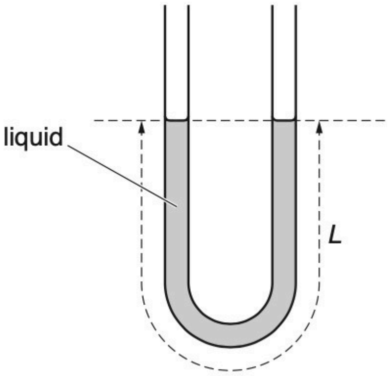


Fig. 3.1

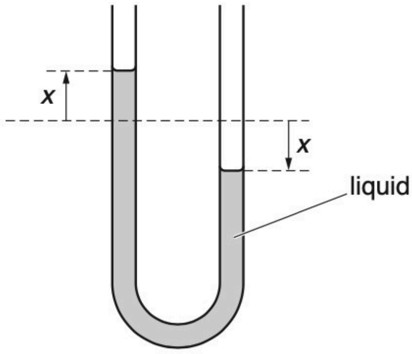


Fig. 3.2

The total length of the column of liquid in the tube is L.

The column of liquid is displaced so that the change in height of the liquid in each arm of the U-tube is x, as shown in Fig. 3.2.

The liquid in the U-tube then oscillates with simple harmonic motion such that the acceleration a of the column is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where g is the acceleration of free fall.

(a) Calculate the period T of oscillation of the liquid column for a column length L of 19.0 cm.

T = ... s [3]

(b) The variation with time t of the displacement x is shown in Fig. 3.3.

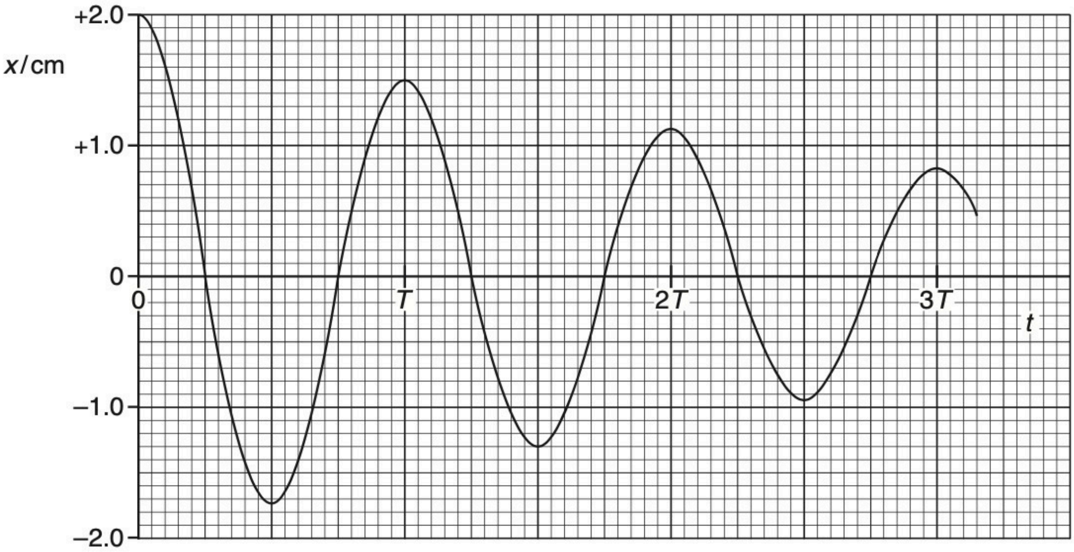


Fig. 3.3

The period of oscillation of the liquid column of mass 18.0 g is T.

The oscillations are damped.

(i) Suggest one cause of the damping.

....

....[1]

(ii) Calculate the loss in total energy of the oscillations during the first 2.5 periods of the oscillations.

energy loss = ... J [3]

18 **(a)** State, by reference to simple harmonic motion, what is meant by *angular frequency*.

....

.... [1]

(b) A thin metal strip is clamped at one end so that it is horizontal.
A load of mass M is attached to its free end. The load causes a displacement s of the end of the strip, as shown in Fig. 2.1.

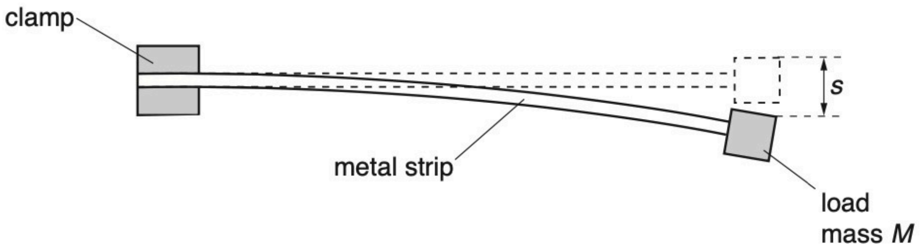


Fig. 2.1

The load is displaced vertically and then released. The load oscillates.
The variation with the acceleration a of the displacement s of the load is shown in Fig. 2.2.

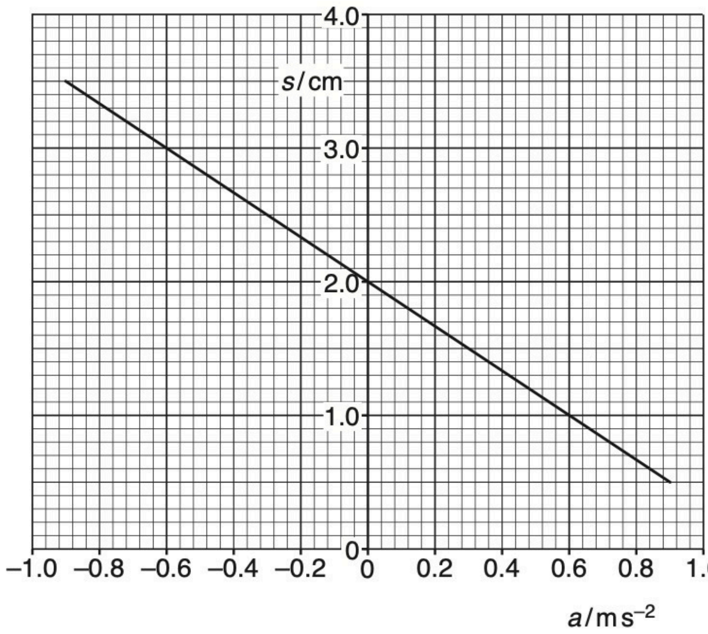


Fig. 2.2

(i) Use Fig. 2.2 to determine

1. the displacement of the load before it is made to oscillate,

displacement = ... cm

2. the amplitude of the oscillations of the load.

amplitude = ... cm
[2]

(ii) Show that the load is undergoing simple harmonic motion.

....

....

....

.... [3]

(iii) Calculate the frequency of oscillation of the load.

frequency = ... Hz [3]

[Total: 9]

19 A spring is hung vertically from a fixed point. A mass M is hung from the other end of the spring, as illustrated in Fig. 3.1.

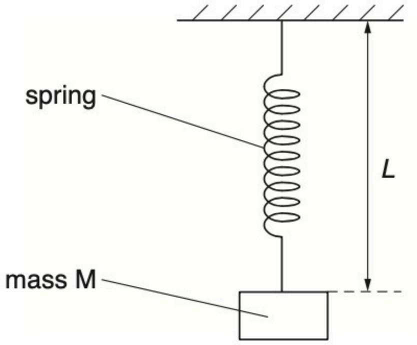


Fig. 3.1

The mass is displaced downwards and then released. The subsequent motion of the mass is simple harmonic.

The variation with time *t* of the length *L* of the spring is shown in Fig. 3.2.

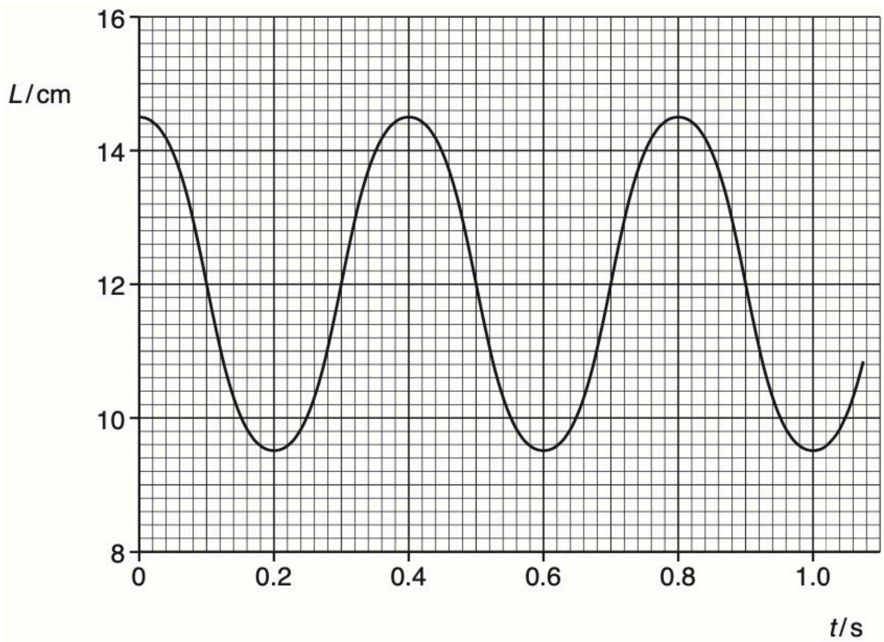


Fig. 3.2

(a) State:

(i) one time at which the mass is moving with maximum speed

time = ... s [1]

(ii) one time at which the spring has maximum elastic potential energy.

shaq

time = ... s [1]

(b) Use data from Fig. 3.2 to determine, for the motion of the mass:

(i) the angular frequency ω

ω = ... rad s^{-1} [2]

(ii) the maximum speed

maximum speed = ... m s^{-1} [2]

(iii) the magnitude of the maximum acceleration.

maximum acceleration = ... m s^{-2} [2]

(c) The mass M is now suspended from two springs, each identical to that in Fig. 3.1, as shown in Fig. 3.3.

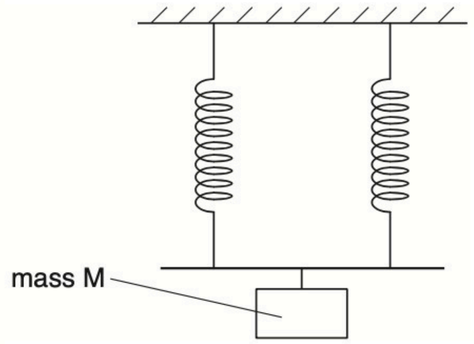


Fig. 3.3

Suggest and explain the change, if any, in the period of oscillation of the mass. A numerical answer is not required.

....

....

....[2]

[Total: 10]

20 A cylindrical tube, sealed at one end, has cross-sectional area A and contains some sand. The total mass of the tube and the sand is M.

The tube floats upright in a liquid of density ρ, as illustrated in Fig. 3.1.

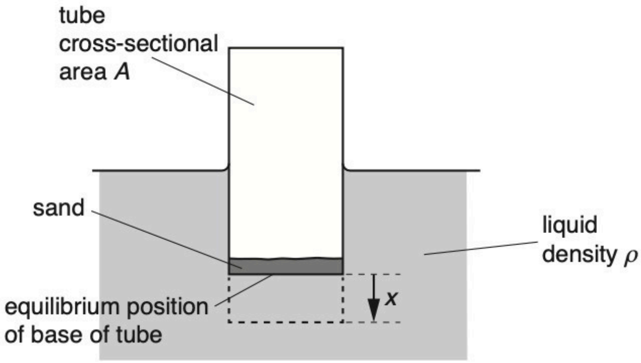


Fig. 3.1

The tube is pushed a short distance into the liquid and then released.

(a) (i) State the two forces that act on the tube immediately after its release.

....

.... [1]

(ii) State and explain the direction of the resultant force acting on the tube immediately after its release.

....

....

.... [2]

(b) The acceleration a of the tube is given by the expression

$$a = -\left(\frac{A\rho g}{M}\right)x$$

where x is the vertical displacement of the tube from its equilibrium position.

Use the expression to explain why the tube undergoes simple harmonic oscillations in the liquid.

....

....

; Ishaq
; .... [2]

(c) For a tube having cross-sectional area A of $4.5\,cm^2$ and a total mass M of 0.17 kg, the period of oscillation of the tube is 1.3 s.

(i) Determine the angular frequency ω of the oscillations.

ω = ... $rad\,s^{-1}$ [2]

(ii) Use your answer in **(i)** and the expression in **(b)** to determine the density ρ of the liquid in which the tube is floating.

ρ = ... $kg\,m^{-3}$ [3]

[Total: 10]

21 A cylinder and piston, used in a car engine, are illustrated in Fig. 3.1. *O/N/10/43/Q3*

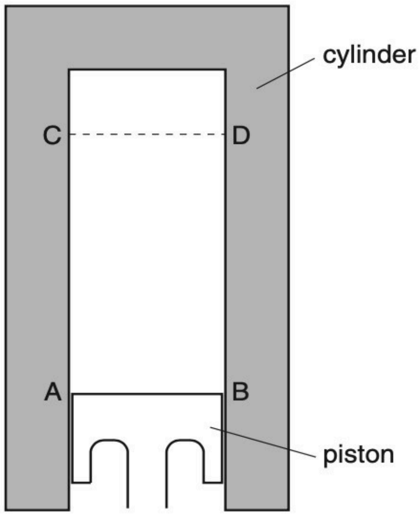


Fig. 3.1

The vertical motion of the piston in the cylinder is assumed to be simple harmonic.
The top surface of the piston is at AB when it is at its lowest position; it is at CD when at its highest position, as marked in Fig. 3.1.

(a) The displacement d of the piston may be represented by the equation

$$d = -4.0 \cos(220t)$$

where d is measured in centimetres.

(i) State the distance between the lowest position AB and the highest position CD of the top surface of the piston.

distance = ... cm [1]

(ii) Determine the number of oscillations made per second by the piston.

number = ... [2]

(iii) On Fig. 3.1, draw a line to represent the top surface of the piston in the position where the speed of the piston is maximum. [1]

(iv) Calculate the maximum speed of the piston.

speed = ... cm s^{-1} [2]

(b) The engine of a car has several cylinders. Three of these cylinders are shown in Fig. 3.2.

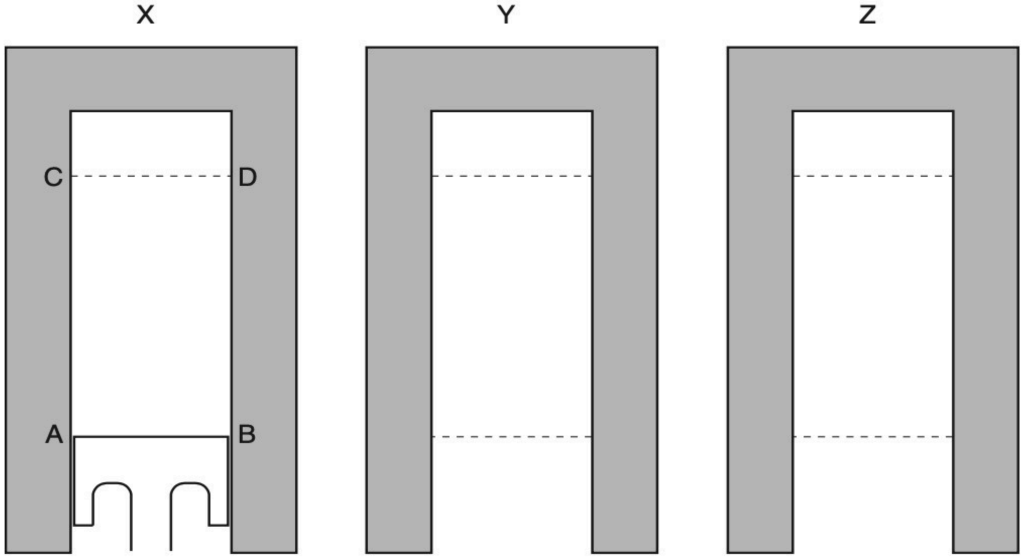


Fig. 3.2

X is the same cylinder and piston as in Fig. 3.1.
Y and Z are two further cylinders, with the lowest and the highest positions of the top surface of each piston indicated.
The pistons in the cylinders each have the same frequency of oscillation, but they are not in phase.
At a particular instant in time, the position of the top of the piston in cylinder X is as shown.

(i) In cylinder Y, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 120° ($\frac{2}{3}\pi$ rad).
Complete the diagram of cylinder Y, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

22 A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

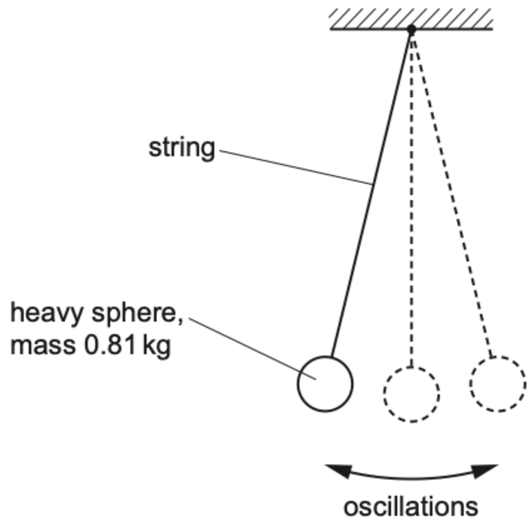


Fig. 4.1

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s.

(a) State what is meant by simple harmonic motion.

....

....

.... [2]

(b) Calculate:

(i) the angular frequency of the oscillations

angular frequency = ... $rad\,s^{-1}$ [2]

(ii) the total energy of the oscillations.

total energy = ... J [2]

(c) The suspended sphere is now lowered into water. The sphere is given a sideways displacement of +0.036 m from its equilibrium position and is then released at time $t = 0$.
The water causes the motion of the sphere to be critically damped.

On Fig. 4.2, sketch the variation of the displacement x of the sphere from its equilibrium position with t from $t = 0$ to $t = 6.0$ s.

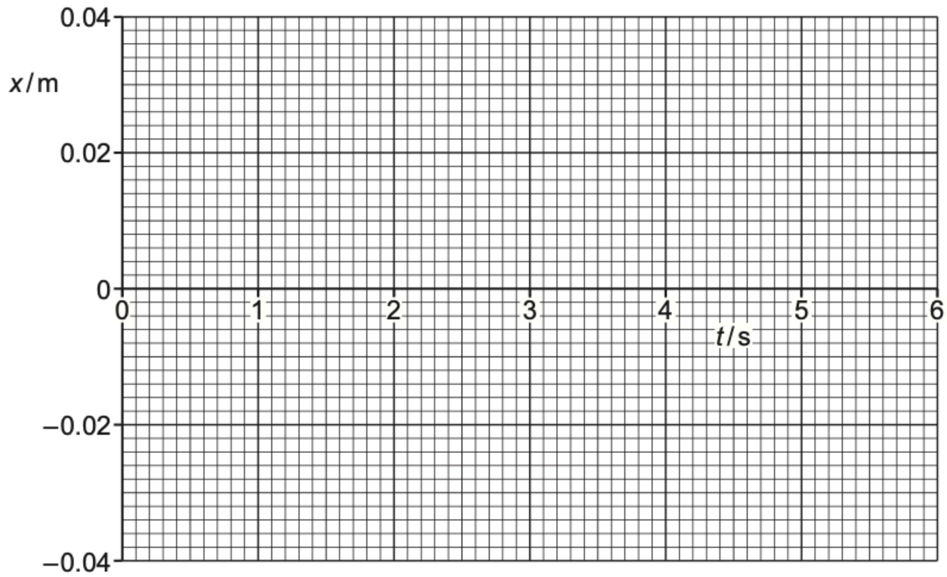


Fig. 4.2

[3]

[Total: 9]

23 A small object of mass 24 g rests on a platform. The platform is attached to an oscillator, as shown in Fig. 3.1.

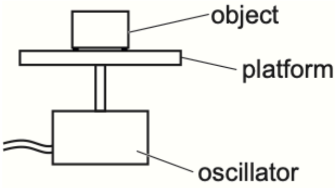


Fig. 3.1

The oscillator moves the platform up and down.

(a) The total energy of the oscillations of the object is 2.2×10^{-4} J.
In one oscillation the object travels a total distance of 14 mm.

Calculate the angular frequency ω of the oscillations.

ω = ... rad s^{-1} [3]

(b) The frequency of the oscillator is fixed, and the amplitude of the oscillations is gradually increased.

(i) Calculate the maximum amplitude of the oscillations so the object does not lose contact with the platform.

amplitude = ... m [2]

(ii) The amplitude of the oscillations is increased so it is greater than the value in **(b)(i)**.

State and explain the position in an oscillation where the object first loses contact with the platform.

....

....

....

.... [2]

24 **(a)** State what is meant by resonance.

....

....

.... [2]

(b) A small ball is held in place using a stretched string. One end of the string is fixed to a wall and the other end is attached to a vibration generator, as shown in Fig. 4.1.

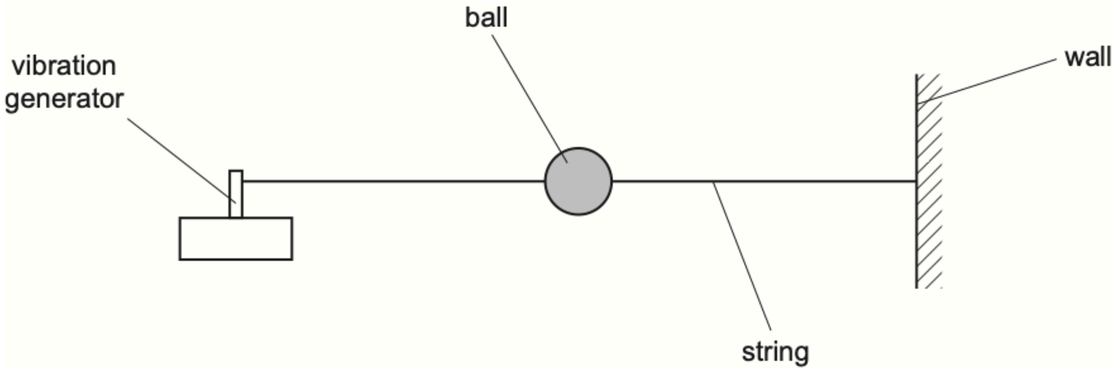


Fig. 4.1

Initially, the vibration generator is switched off.

A student displaces the ball vertically and then releases it. Fig. 4.2 shows the variation of the displacement of the ball with time after it is released.

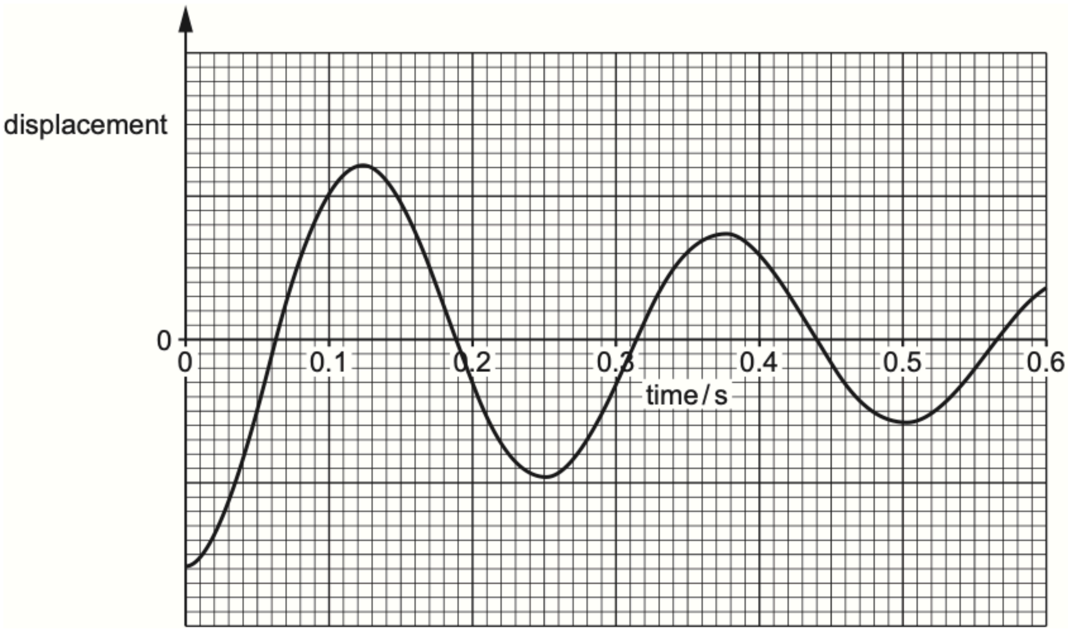


Fig. 4.2

(i) State the name of the phenomenon illustrated by the decrease in the amplitude of the oscillations in Fig. 4.2.

.... [1]

(ii) Explain the decrease with time of the amplitude of the oscillations of the ball.

....

....

.... [2]

(iii) Determine the frequency of the oscillations of the ball.

frequency = ... Hz [1]

(c) The vibration generator in **(b)** is switched on and its frequency *f* of vibration is gradually increased from 0 to 10 Hz.

On Fig. 4.3, sketch the variation with *f* of the amplitude of the oscillations of the ball.

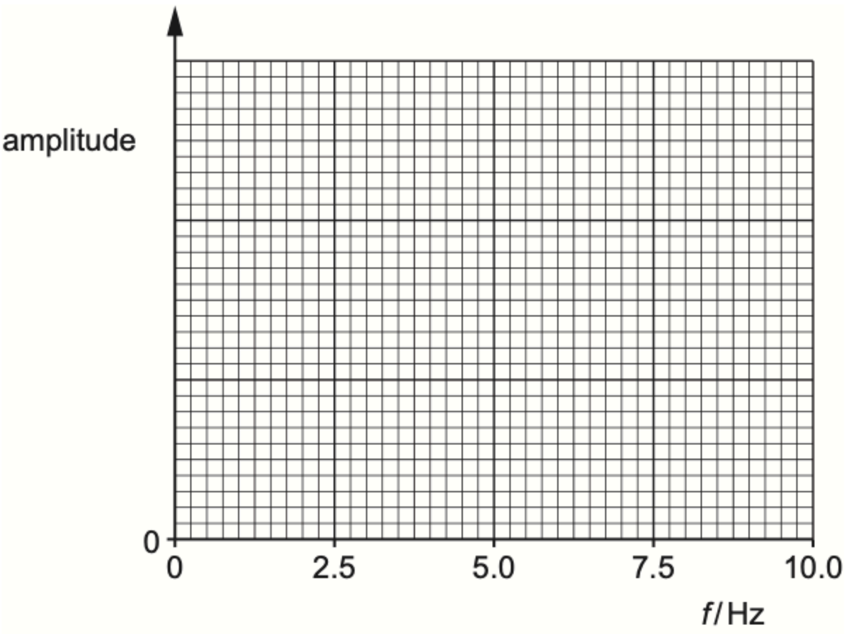


Fig. 4.3

[2]

[Total: 8]

25 A block of mass *m* oscillates vertically on a spring, as shown in Fig. 4.1.

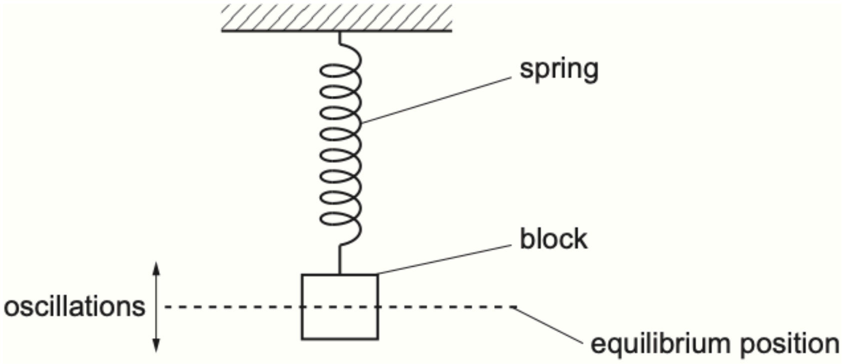


Fig. 4.1

The acceleration *a* of the block varies with displacement *x* from its equilibrium position, as shown in Fig. 4.2.

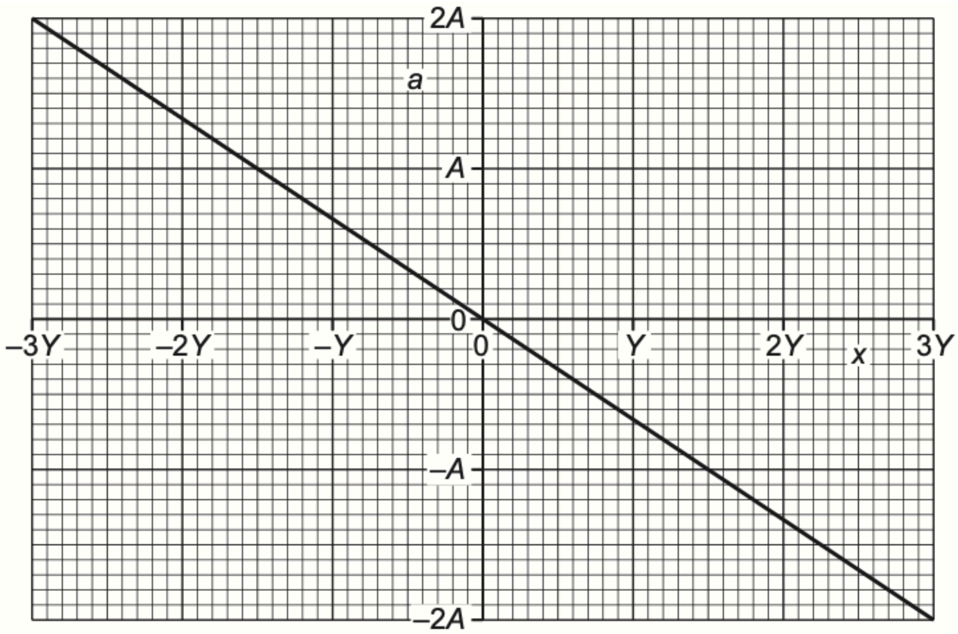


Fig. 4.2

The amplitude of the oscillations is $3Y$ and the maximum acceleration is $2A$.

(a) Explain how Fig. 4.2 shows that the oscillations of the block are simple harmonic.

....

....

.... [2]

(b) Deduce expressions, in terms of some or all of m, A and Y, for:

(i) the angular frequency ω of the oscillations

ω = ... [1]

(ii) the maximum speed v_0 of the oscillations

v_0 = ... [2]

(iii) the energy E of the oscillations.

E = ... [2]

(c) The period of the oscillations is 0.75 s and the value of $3Y$ is 1.8 cm.

Determine an expression for x in terms of time t, where x is in cm and t is in seconds.

x = ... [2]

[Total: 9]

Question		Answer	Marks
1	(a)(i)	$\omega = 2\pi f$	C1
		$f = 9.7 / 2\pi$ $= 1.5$ Hz	A1
	(a)(ii)	amplitude = $\sqrt{(11.6)}$ = 3.4 cm	A1
	(a)(iii)	$a_0 = \omega^2 x_0$	C1
		$= 9.7^2 \times 3.4 \times 10^{-2}$ $= 3.2$ m s^{-2}	A1
	(b)	sketch: straight line through the origin with negative gradient	B1
		line with negative gradient passing through (+3.4, $-a_0$) and (–3.4, $+a_0$)	B1
		line with ends at $x = \pm$ 3.4 cm and $a = \pm a_0$	B1
	(c)	sum of potential energy and kinetic energy is constant	B1
		at maximum displacement, kinetic energy is zero **or** at maximum displacement, potential energy is maximum	B1
		at zero displacement, kinetic energy is maximum **or** at zero displacement, potential energy is minimum	B1

Question		Answer	Marks
22	(a)	P: total energy Q: potential energy R: kinetic energy	B2
	(b)	$E = ½m\omega^2 x_0^2$ **or** $E = ½mv_0^2$ **and** $v_0 = \omega x_0$	C1
		$6.4 \times 10^{-3} = \frac{1}{2} \times 0.130 \times \omega^2 \times 0.015^2$ ($\omega^2 = 438$) ($\omega = 20.9$)	C1
		$T = 2\pi / \omega$	C1
		$= 2\pi / 20.9$ $= 0.30$ s	A1
	(c)(i)	resistive forces	B1
	(c)(ii)	0.92^6	C1
		decrease in energy = 6.4 – (6.4 × 0.92^6) = 2.5 mJ	A1
	(c)(iii)	light damping because the amplitude of oscillations gradually reduces **or** light damping because the system still oscillates	B1

Question		Answer	Marks
3	(a)(i)	$x_0 = 8.0$ cm	A1
	(a)(ii)	$\omega = 2\pi / T$	C1
		$= 2\pi / 4.0 = 1.6$ rad s^{-1}	A1
	(a)(iii)	$E = ½m\omega^2 x_0^2$	C1
		$= ½ \times 36 \times 1.6^2 \times 0.080^2$	C1
		$= 0.29$ J	A1
	4(b)	dome-shaped curve, starting and ending at $E_K = 0$	B1
		maximum E_K shown as 0.29 J	B1
		position of peak shown at h = 10.0 cm	B1
		line intercepts h-axis at h = 2.0 cm and at h = 18.0 cm	B1

Question	Answer	Marks
4(a)	$a = -\omega^2 x$	M1
	a = acceleration, x = displacement from equilibrium position and ω = angular frequency	A1
(b)(i)	$x_0 = 0.12$ m	A1
(b)(ii)	$v = \omega\sqrt{(x_0^2 - x^2)}$ two (x, v) pairs correctly read from Fig. 3.2 (one may be (x_0, 0) or value of x_0 from **(i)**)	C1
	e.g. $0.20 = \omega\sqrt{(0.12^2 - 0)}$ leading to $\omega = 1.7$ rad s^{-1}	A1
(b)(iii)	$E = ½M\omega^2 x_0^2$	C1
	$0.050 = ½ \times M \times 1.67^2 \times 0.12^2$ $M = 2.5$ kg	A1
	or	
	$(E_K)_{max} = ½Mv_0^2$	(C1)
	$0.050 = ½ M \times 0.20^2$ $M = 2.5$ kg	(A1)
(c)(i)	loss of (total) energy (of system)	B1
	due to resistive forces	B1
(c)(ii)	closed loop surrounding the origin with maximum x at ± 0.060 m passing through $v = 0$	B1
	maximum velocity shown as ± 0.10 m s^{-1} passing through $x = 0$	B1

Question	Answer	Marks
5(a)	oscillations (of object) at maximum amplitude	B1
	when driving frequency equals natural frequency (of object)	B1
(b)(i)	$T = 2\pi / \omega$	C1
	$= 2\pi / 5.0\pi$ $= 0.40$ s	A1
(b)(ii)	displacement scale labelled –1.0, –0.5, (0), 0.5, 1.0 on the 2 cm tick marks	B1
	t scale labelled 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 on the 2 cm tick marks	B1
(b)(iii)	$\phi = 2\pi\Delta t / T$ $= 2\pi \times 0.10 / 0.40$ **or** $2\pi \times 0.30 / 0.40$	C1
	$= 1.6$ rad **or** 4.7 rad	A1

Question	Answer	Marks
6(a)	straight line through origin shows that a is proportional to x	B1
	negative gradient shows that a is in opposite direction to x	B1
(b)(i)	$a_0 = \omega^2 x_0$ **or** $a = -\omega^2 x$ **or** $\omega^2 = -$ gradient	C1
	$\omega = \sqrt{(0.40 / 0.050)}$ $= 2.8$ rad s^{-1}	A1
(b)(ii)	$k = \omega^2 L$ $= 2.8^2 \times 1.24$	C1
	$= 9.7$ m s^{-2}	A1
(c)	(increasing L causes) ω to decrease **or** energy ($= ½ m\omega^2 x_0^2$) $= ½ mkx_0^2 / L$ (and L increases)	M1
	so amplitude increases	A1

7

Question	Answer	Marks
3(a)	upthrust, weight	B1
3(b)	upthrust greater than weight so (resultant force is) upwards	B1
(c)(i)	*A*, *g* and ρ all constant so $F \propto x$	B1
	minus sign means *F* and *x* are in opposite directions	B1
(c)(ii)	$(a = \frac{F}{m}$ so) $a = (-)\frac{Ag\rho x}{m}$	M1
	so $\omega^2 = \frac{Ag\rho}{m}$ hence $\omega = \sqrt{\frac{Ag\rho}{m}}$	A1
(d)(i)	damping due to viscous forces	B1
(d)(ii)	$(E =) \frac{1}{2} m\omega^2 x_0^2$	C1
	$\omega^2 = (-)$ gradient	C1
	$(E =) \frac{1}{2} m\omega^2 (x_1^2 - x_2^2)$ $= \frac{1}{2} \times 0.57 \times (2.3/0.020)(0.020^2 - 0.016^2)$ $= 4.7 \times 10^{-3}$ J	A1

8

(a)	acceleration and displacement are in opposite directions
(b)(i)	$F = kx$ $= 8.0 \times (0.060 - 0.048)$ *or* $8.0 \times (0.060 + 0.048)$ or 8.0×0.012 *or* 8.0×0.108
	$\Sigma F = (8.0 \times 0.012) - (8.0 \times 0.108) = 0.77 N$ or $\Sigma F = 0.864 - 0.096 = 0.77 N$
(b)(ii)	$a = \frac{F}{m}$ $= \frac{0.77}{0.25}$ $= 3.1 ms^{-2}$
(b)(iii)	$a = -\omega^2 x$ $\omega = \sqrt{\frac{3.1}{0.048}}$ $\omega = 8.04$
	$T = 2\pi / \omega$
	$T = 2\pi / 8.04$ $= 0.78$ s
(b)(iv)	(resultant) force halved and distance halved
	same T

9	(a)	acceleration in opposite direction to displacement shown by – sign
		g / L is constant
		(so) acceleration is (directly) proportional to displacement
	(b)	$\omega^2 = g / L$
		$\omega = 2\pi / T$ **or** $\omega = 2\pi f$ **and** $f = 1 / T$
		$(2\pi / T)^2 = 9.81 / 0.18$ $T = 0.85$ s
	(c)	energy $\propto x_0^2$
		(after 3 cycles,) amplitude $= (0.94)^3 x_0$ $= 0.83 x_0$
		ratio final energy / initial energy $= 0.83^2$ $= 0.69$

10	(a)	acceleration (directly) proportional to displacement
		acceleration is in opposite direction to displacement
	(b)	$\omega^2 = 2k / m$ **and** $\omega = 2\pi f$
		$(2\pi f)^2 = (2 \times 130) / 0.84$
		$f = 2.8$ Hz
	(c)(i)	resonance
	(c)(ii)	oscillator supplies energy (continuously)
		energy of trolley constant so energy must be dissipated **or** without loss of energy the amplitude would continuously increase

11	(a)	acceleration (directly) proportional to displacement
		acceleration in opposite <u>direction</u> to displacement **or** acceleration (directed) towards equilibrium position
	(b)	$v = \omega(x_0^2 - x^2)^{½}$ **and** $\omega = 2\pi f$ **or** $v_0 = x_0\omega$ **and** $\omega = 2\pi f$
		substitution of any correct point from graph, e.g. for $x = 0$: $0.25 = 2\pi f \times 8.8 \times 10^{-2}$
		$f = 0.45$ Hz
	(c)	$1 / 0.45 = 2\pi \times (L / 9.81)^{½}$
		$L = 1.2$ m
	(d)	ellipse about the origin with same intercepts on x-axis
		ellipse about the origin crossing v-axis inside original loop

12	(a)(i)	distance from a (reference) point in a given direction
	(a)(ii)	line is not straight **or** gradient is not constant
	(b)(i)	0.85–0.90 cm
	(b)(ii)	$a = -(2\pi f)^2 x$
		e.g. $1.2 = 4\pi^2 \times f^2 \times (0.90 \times 10^{-2})$
		$f = 1.8$ Hz
	(c)	complete circle/ellipse enclosing the origin
		closed shape passing through $(0, \pm v_0)$ and $(\pm x_0, 0)$

13	(a)	acceleration (directly) proportional to displacement
		acceleration is in opposite <u>direction</u> to displacement **or** acceleration is (directed) towards a fixed point
	(b)(i)	zero
	(b)(ii)	E_T is maximum potential energy = mgh $E_T = 94 \times 10^{-3} \times 9.81 \times 0.90 \times 10^{-2}$
		$= 8.3 \times 10^{-3}$ J
	(b)(iii)	$E_{MAX} = \frac{1}{2} m v_0^2$ **and** $v_0 = \omega x_0$ **or** $E_{MAX} = \frac{1}{2} m(\omega x_0)^2$
		$8.3 \times 10^{-3} = \frac{1}{2} \times 94 \times 10^{-3} \times \omega^2 \times (12.7 \times 10^{-2})^2$ …leading to $\omega = 3.3$ rad s^{-1}
	(c)	$T = 2\pi / \omega$
		$2\pi / 3.3 = 2\pi \times (L / 9.81)^{\frac{1}{2}}$
		$L = 0.90$ m

14	(a)	acceleration proportional to displacement
		acceleration <u>directed</u> towards fixed point **or** displacement and acceleration in opposite <u>directions</u>
	(b)(i)	**1.** amplitude decreases gradually so light damping **or** oscillations continue so light damping
		2. loss of energy
		due to friction in wheels **or** due to friction between wheels and surface (during slipping) **or** due to air resistance (on trolley)
	(b)(ii)1.	$\omega^2 = 2k / m$
		$= (2 \times 230) / 0.950$
		$\omega = 22$ rad s^{-1}
	(b)(ii)2.	$T = 2\pi / \omega$
		$T = (2\pi / 22) = 0.286$ s time $= 1.5T$ $= 0.43$ s

15		
(a)(i)	straight line through origin indicates acceleration ∝ displacement
	negative gradient shows acceleration and displacement are in opposite directions
(a)(ii)	$a = -\omega^2 y$ **and** $\omega = 2\pi f$ $4.5 = (2\pi \times f)^2 \times 8.0 \times 10^{-3}$ *(or other valid read-off)*
	$f = 3.8$ Hz
(b)(i)	maximum displacement upwards/above rest/above the equilibrium position
(b)(ii)	(just leaves plate when) acceleration = $9.81\ \text{m s}^{-2}$
	$9.81 = (2\pi \times 3.8)^2 \times y_0$ **or** $9.81 = 563 \times y_0$
	amplitude = 17 mm

16

(a) acceleration/force proportional to displacement (from fixed point)

acceleration/force and displacement in opposite directions

(b) <u>maximum</u> displacements/accelerations are different

graph is curved/not a straight line

(c) **(i)** $\omega = 2\pi / T$ and $T = 0.8$ s

$\omega = 7.9\ \text{rad s}^{-1}$

(ii) $a = (-)\omega^2 x$
$= 7.85^2 \times 1.5 \times 10^{-2}$

$= 0.93\ \text{m s}^{-2}$ or $0.94\ \text{m s}^{-2}$

(iii) $\Delta E = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

$= \frac{1}{2} \times 120 \times 10^{-3} \times 7.85^2 \times \{(1.5 \times 10^{-2})^2 - (0.9 \times 10^{-2})^2\}$

$= 5.3 \times 10^{-4}$ J

17 (a)	$\omega^2 = 2g/L$	
	$T = 2\pi/\omega$	
	$\omega^2 = (2 \times 9.81)/0.19$ $\omega = 10.2$ (rad s^{-1}) $T = 2\pi/10.2$ $= 0.62$ s	
(b)(i)	e.g. viscosity of liquid/friction within the liquid/viscous drag/friction between walls of tube and liquid	
(b)(ii)	(maximum) KE = $\frac{1}{2}mv_0^2$ **and** $v_0 = \omega x_0$ **or** energy = $\frac{1}{2}m\omega^2 x_0^2$	
	change $= \frac{1}{2} \times 18 \times 10^{-3} \times 103 \times [(2.0 \times 10^{-2})^2 - (0.95 \times 10^{-2})^2]$	
	$= 2.9 \times 10^{-4}$ J	

18 (a)	(angular frequency =) $2\pi \times$ frequency **or** 2π/period
(b)(i)	**1.** displacement = 2.0 cm
	2. amplitude = 1.5 cm
(b)(ii)	reference to displacement of oscillations **or** displacement from equilibrium position **or** displacement from 2.0 cm
	straight line indicates acceleration $\propto$ displacement
	negative gradient shows acceleration and displacement are in opposite directions
(b)(iii)	$\omega^2 = (-)1$/gradient **or** $\omega^2 = (-)\Delta a/\Delta s$ **or** $a = (-)\omega^2 x$ <u>and</u> correct value of x
	= e.g. (1.8 / 0.03) or (0.9 / 0.015) or (1.2 / 0.02) etc. **or** $0.9 = \omega^2 \times 0.015$ = 60
	$f = \sqrt{60}/2\pi$ = 1.2 Hz

19		
	(a)(i)	0.10 s **or** 0.30 s **or** 0.50 s **or** 0.70 s **or** 0.90 s
	(a)(ii)	0 **or** 0.40 s **or** 0.80 s
	(b)(i)	$\omega = 2\pi / T$
		$= 2\pi / 0.40$ $= 16\ \text{rad s}^{-1}$
	(b)(ii)	$v_0 = \omega x_0$
		$= 15.7 \times 2.5 \times 10^{-2}$ $= 0.39\ \text{m s}^{-1}$
		or
		tangent drawn at steepest part and working to show attempted calculation of gradient
		leading to $v_0 = 0.39\ \text{m s}^{-1}$ (*allow ± 0.15 m s⁻¹*)
	(b)(iii)	$a_0 = \omega^2 x_0$
		$a_0 = (15.7^2 \times 2.5 \times 10^{-2})$ $= 6.2\ \text{m s}^{-2}$
		or
		$a_0 = \omega v_0$
		$a_0 = 15.7 \times 0.39$ $= 6.2\ \text{m s}^{-2}$
	(c)	period is shorter/lower
		Any one from: • greater spring constant/stiffness • (restoring) force is greater (for any given extension) • acceleration is greater (for any given extension) • greater energy/maximum speed (for a given amplitude)

20		
(a)(i)	mention of upthrust and weight	
(a)(ii)	upthrust is greater than the weight	
	(resultant force is) upwards	
(b)	*A*, ρ, *g and M* are constant	
	either acceleration $\propto$ – displacement *or* acceleration $\propto$ displacement <u>and</u> (– sign indicates) *a* and *x* in opposite directions	
(c)(i)	*either* $\omega = 2\pi / T$ *or* $\omega = 2\pi f$ <u>and</u> $f = 1 / T$	
	$\omega = 2\pi / 1.3$ $= 4.8 \text{ rad s}^{-1}$	
(c)(ii)	$\omega^2 = A\rho g / m$	
	$4.83^2 = (4.5 \times 10^{-4} \times \rho \times 9.81) / 0.17$	
	$\rho = 900 \text{ kg m}^{-3}$	

21 **(a)** **(i)** 8.0 cm

(ii) $2\pi f = 220$
$f = 35$ (*condone unit*)

(iii) line drawn mid-way between AB and CD (*allow ±2 mm*)

(iv) $v = \omega a$
$= 220 \times 4.0$
$= 880 \text{ cm s}^{-1}$

(b) **(i)** **1.** line drawn 3 cm above AB (*allow ±2 mm*)
2. arrow pointing upwards

(ii) **1.** line drawn 3 cm above AB (*allow ±2 mm*)
2. arrow pointing downwards

(iii) $v = \omega\sqrt{(a^2 - x^2)}$
$= 220 \times \sqrt{(4.0^2 - 2.0^2)}$
$= 760 \text{ cm s}^{-1}$
(*incorrect value for x, 0/2 marks*)

22 (a)		
	(motion in which) acceleration is (directly) proportional to displacement	**B1**
	(motion in which): acceleration is (always) in the opposite <u>direction</u> to displacement **or** acceleration is (always) directed towards a fixed point	**B1**
4(b)(i)	$\omega = 2\pi / T$	**C1**
	$\omega = 2\pi / 3.0$ $= 2.1 \text{ rad s}^{-1}$	**A1**
4(b)(ii)	$E = \frac{1}{2} m\omega^2 x_0^2$	**C1**
	$= \frac{1}{2} \times 0.81 \times 2.1^2 \times 0.036^2$ $= 2.3 \times 10^{-3}$ J	**A1**
4(c)	sketch: line starting at (0, 0.036) and not reaching $x = \pm 0.036$ m at any other time	**B1**
	smooth curve, with no sudden changes in gradient, showing continuously decreasing magnitude of *x* from maximum displacement at *t* = 0 to final displacement of zero where the gradient is also zero	**B1**
	displacement reaches final value of zero between *t* = 0.75 s and *t* = 3.0 s at the latest	**B1**

23 a)	$E = \frac{1}{2} m\omega^2 x_o^2$	**C1**
	$2.2 \times 10^{-4} = \frac{1}{2} \times 24 \times 10^{-3} \times (14 \times 10^{-3} / 4)^2 \times \omega^2$	**C1**
	$\omega = 39\ \text{rad s}^{-1}$	**A1**
3(b)(i)	use of acceleration = $9.81\ \text{m s}^{-2}$ $x_o = 9.81 / 39^2$	**C1**
	$= 6.4 \times 10^{-3}$ m	**A1**
3(b)(ii)	at top of oscillation	**B1**
	any one point from: where the downward acceleration first exceeds free-fall acceleration where the greatest downwards acceleration occurs where the resultant force is the maximum downwards where the contact force is a minimum	**B1**

24 a)	oscillation (of object) at maximum amplitude	**B1**
	when driving frequency = natural frequency (of system)	**B1**
4(b)(i)	light damping	**B1**
4(b)(ii)	oscillations (of ball) lose energy	**B1**
	(due to) resistive forces (acting on ball)	**B1**
4(b)(iii)	frequency = 1 / 0.25 = 4.0 Hz	**A1**
4(c)	curve showing a maximum amplitude at a single non-zero frequency	**B1**
	single maximum amplitude shown at 4.0 Hz	**B1**

25 (a)	straight line through the origin shows that *a* is proportional to *x*	**B1**
	negative gradient shows that *a* and *x* are (always) in opposite directions	**B1**
4(b)(i)	$a = -\omega^2 x$ $\omega = \sqrt{(2A / 3Y)}$	**A1**
4(b)(ii)	$v_0 = \omega x_0$	**C1**
	$= 3Y \times \sqrt{(2A / 3Y)}$ $= \sqrt{(6AY)}$	**A1**
4(b)(iii)	$E = \frac{1}{2} m\omega^2 x_0^2$	**C1**
	$= \frac{1}{2} m \times (2A / 3Y) \times (3Y)^2$ $= 3mAY$	**A1**
4(c)	$\omega = 2\pi / T$ $(= 2\pi / 0.75)$	**C1**
	$x = 1.8 \sin (8.4\,t)$	**A1**