

Circular Motion

A-Level Worksheet

Q1. (a) With reference to velocity and acceleration, describe uniform circular motion.

.....
.....
..... [2]

(b) Two cars are moving around a horizontal circular track. One car follows path X and the other follows path Y, as shown in Fig. 1.1.

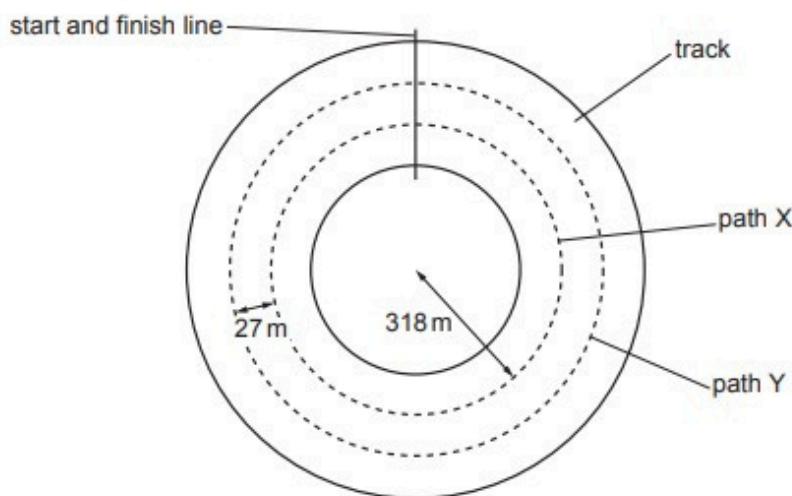


Fig. 1.1 (not to scale)

The radius of path X is 318 m. Path Y is parallel to, and 27 m outside, path X. Both cars have mass 790 kg. The maximum lateral (sideways) friction force F that the cars can experience without sliding is the same for both cars.

(i) The maximum speed at which the car on path X can move around the track without sliding is 94 ms^{-1} .

Calculate F .

$$F = \dots \text{ N} [2]$$

(ii) Both cars move around the track. Each car has the maximum speed at which it can move without sliding.

Complete Table 1.1, by placing one tick in each row, to indicate how the quantities indicated for the car on path Y compare with the car on path X.

Table 1.1

	Y less than X	Y same as X	Y greater than X
centripetal acceleration			
maximum speed			
time taken for one lap of the track			

[3]

[Total: 7]

Q2. (a) State what is meant by *centripetal* acceleration.

.....
.....
.....

[1]

(b) An unpowered toy car moves freely along a smooth track that is initially horizontal. The track contains a vertical circular loop around which the car travels, as shown in Fig. 1.1.

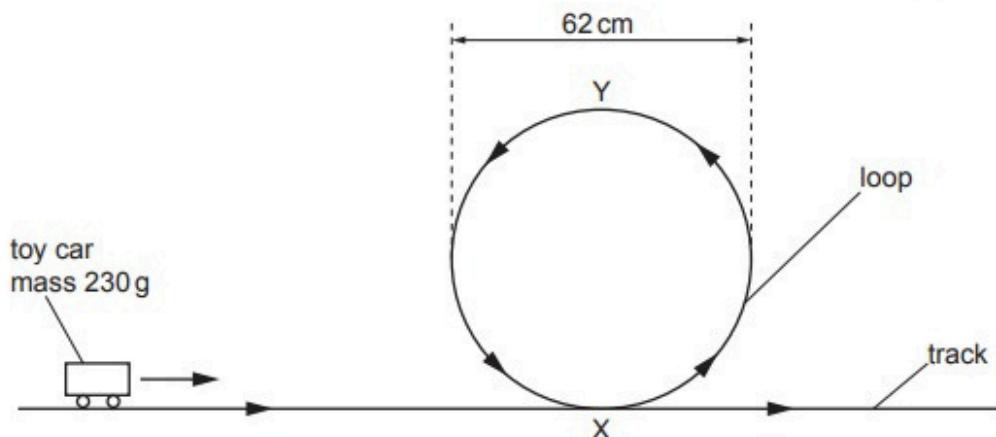


Fig. 1.1

The mass of the car is 230 g and the diameter of the loop is 62 cm. Assume that the resistive forces acting on the car are negligible.

(i) State what happens to the magnitude of the centripetal acceleration of the car as it moves around the loop from X to Y.

.....
.....

[1]

(ii) Explain, if the car remains in contact with the track, why the centripetal acceleration of the car at point Y must be greater than 9.8 ms^{-2} .

.....
.....
.....

[2]

(c) The initial speed at which the car in **(b)** moves along the track is 3.8 m s^{-1} .

Determine whether the car is in contact with the track at point Y. Show your working.

[3]

(d) Suggest, with a reason but without calculation, whether your conclusion in **(c)** would be different for a car of mass 460g moving with the same initial speed.

.....
.....
.....

[1]

[Total: 8]

Q3. (a) Define the *radian*.

.....
.....
.....

[2]

(b) A telescope gives a clear view of a distant object when the angular displacement between the edges of the object is at least 9.7×10^{-6} rad.

(i) The Moon is approximately 3.8×10^5 km from Earth.

Estimate the minimum diameter of a circular crater on the Moon's surface that can be seen using the telescope.

diameter = km [2]

(ii) Suggest why craters of the same diameter as that calculated in (i) but on the surface of Mars are not visible using this telescope.

.....
.....
.....

[2]

Q4. A large bowl is made from part of a hollow sphere.

A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius, as shown in Fig. 2.1.

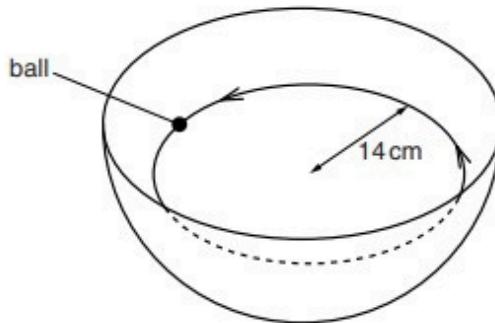


Fig. 2.1

The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball, as shown in Fig. 2.2.

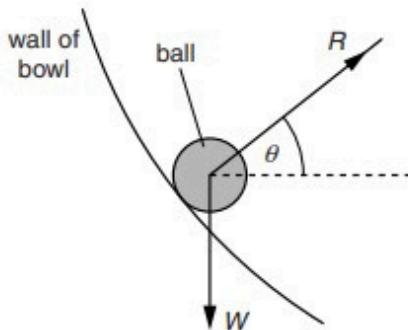


Fig. 2.2

The normal reaction force R is at an angle θ to the horizontal.

(a) (i) By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by the expression

$$W = F \tan \theta.$$

[2]

(ii) State the significance of the force F for the motion of the ball in the bowl.

.....
..... [1]

(b) The ball moves in a circular path of radius 14cm. For this radius, the angle θ is 28° .

Calculate the speed of the ball.

speed = ms^{-1} [3]

Q5. (a) Define the radian.

.....
.....
.....

[2]

(b) A stone of weight 3.0N is fixed, using glue, to one end P of a rigid rod CP, as shown in Fig. 1.1.

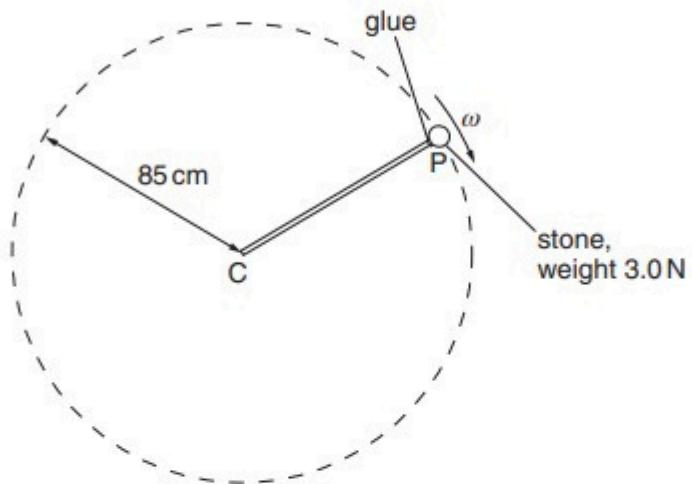


Fig. 1.1

The rod is rotated about end C so that the stone moves in a vertical circle of radius 85 cm.

The angular speed ω of the rod and stone is gradually increased from zero until the glue snaps. The glue fixing the stone snaps when the tension in it is 18N.

For the position of the stone at which the glue snaps,

- (i) on the dotted circle of Fig. 1.1, mark with the letter S the position of the stone, [1]
- (ii) calculate the angular speed ω of the stone.

angular speed = rad s⁻¹ [4]

Q6. A vertical peg is attached to the edge of a horizontal disc of radius r , as shown in Fig. 4.1.

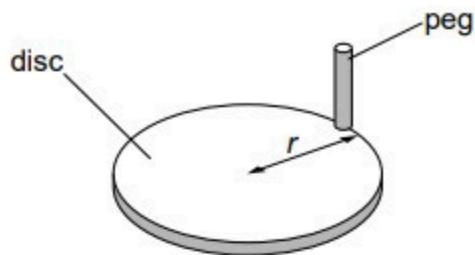


Fig. 4.1

The disc rotates at constant angular speed ω . A horizontal beam of parallel light produces a shadow of the peg on a screen, as shown in Fig. 4.2.

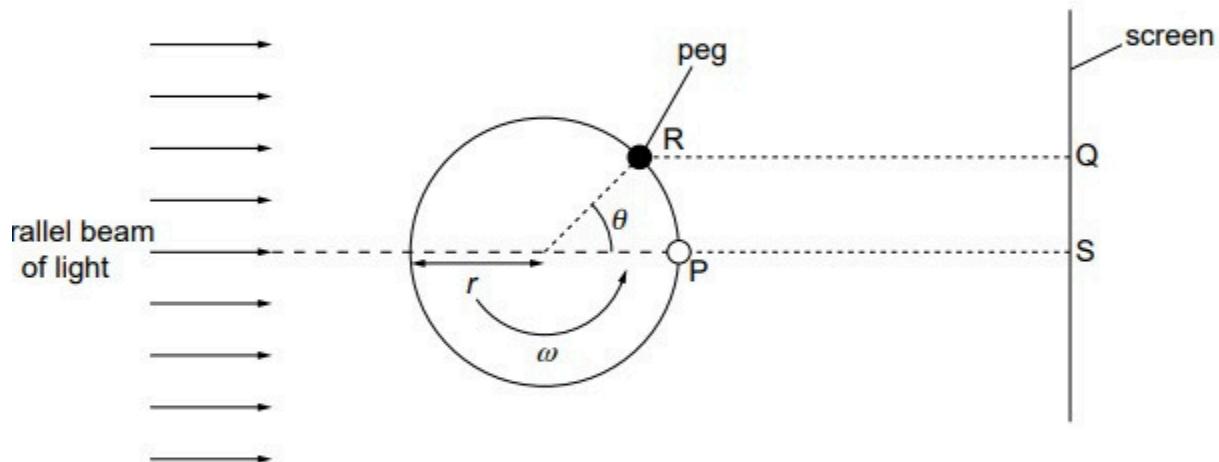


Fig. 4.2 (plan view)

At time zero, the peg is at P, producing a shadow on the screen at S.

At time t , the disc has rotated through angle θ . The peg is now at R, producing a shadow at Q.

(a) Determine,

(i) in terms of ω and t , the angle θ ,

..... [1]

(ii) in terms of ω , t and r , the distance SQ.

..... [1]

(b) Use your answer to (a)(ii) to show that the shadow on the screen performs simple harmonic motion.

.....
.....
.....

[2]

(c) The disc has radius r of 12 cm and is rotating with angular speed ω of 4.7 rad s^{-1} .

Determine, for the shadow on the screen,

(i) the frequency of oscillation,

frequency = Hz [2]

(ii) its maximum speed.

speed = cm s^{-1} [2]

Q7. (a) (i) Define the *radian*.

.....
.....
.....

[2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

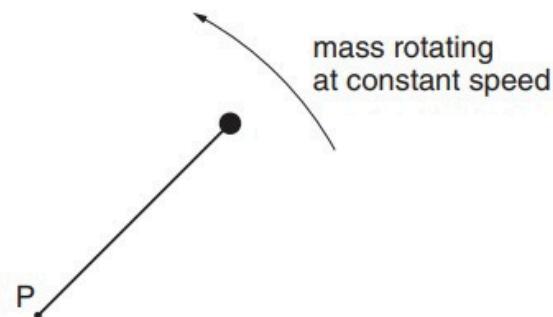


Fig. 1.1

Explain what is meant by the *angular speed* about point P of the mass.

.....
.....
.....

[2]

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

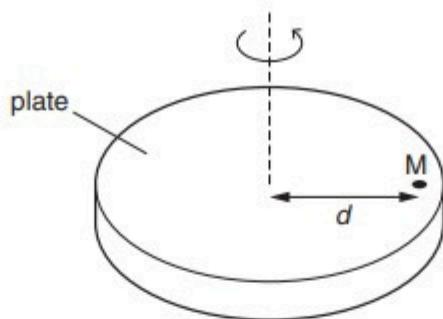


Fig. 1.2

A small mass M is placed on the plate, a distance d from the axis of rotation. The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force F between the plate and the mass is given by the expression

$$F = 0.72W,$$

where W is the weight of the mass M.

The distance d is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working.

number = [5]

(c) The plate in (b) is covered, when stationary, with mud.

Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

.....
.....
.....

[2]

Q8. (a) Explain

(i) what is meant by a *radian*,

.....
.....
.....

[2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad.

.....
.....

[1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 1.1.

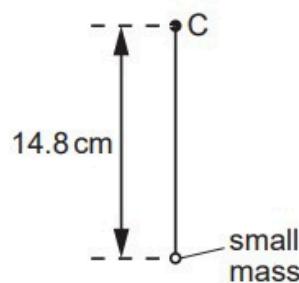


Fig. 1.1

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 1.2.

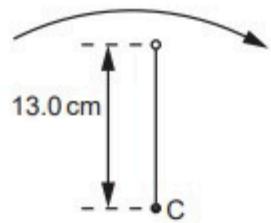


Fig. 1.2

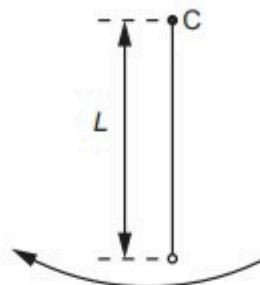


Fig. 1.3

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1} .

[2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 1.3.

Calculate the length L of the cord, assuming it obeys Hooke's law.

$$L = \dots \text{ cm} \quad [4]$$

Q9 A steel sphere of mass 0.29 kg is suspended in equilibrium from a vertical spring. The centre of the sphere is 8.5 cm from the top of the spring, as shown in Fig. 2.1.

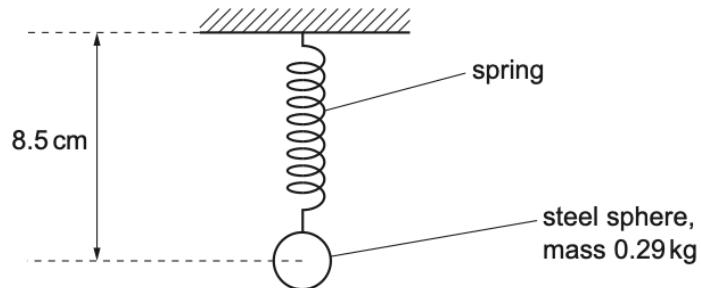


Fig. 2.1

The sphere is now set in motion so that it is moving in a horizontal circle at constant speed, as shown in Fig. 2.2.

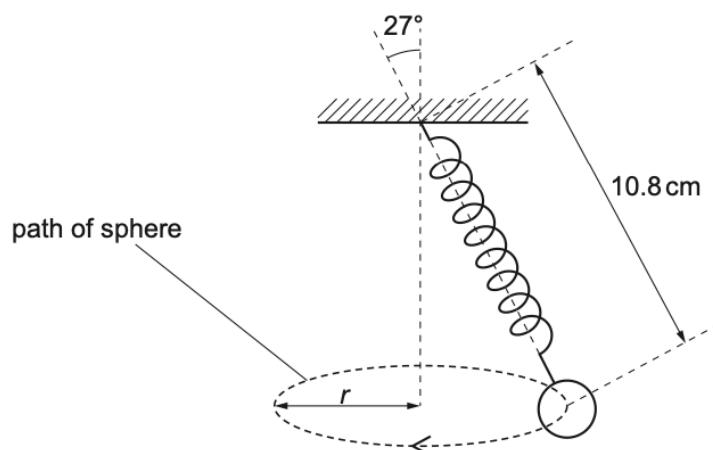


Fig. 2.2

The distance from the centre of the sphere to the top of the spring is now 10.8 cm.

(a) Explain, with reference to the forces acting on the sphere, why the length of the spring in Fig. 2.2 is greater than in Fig. 2.1.

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.....
.....
.....
.....

[3]

(b) The angle between the linear axis of the spring and the vertical is 27° .

(i) Show that the radius r of the circle is 4.9 cm.

[1]

(ii) Show that the tension in the spring is 3.2 N.

[2]

(iii) The spring obeys Hooke's law.

Calculate the spring constant, in N cm^{-1} , of the spring.

spring constant = N cm^{-1} [2]

(c) (i) Use the information in **(b)** to determine the centripetal acceleration of the sphere.

centripetal acceleration = ms^{-2} [2]

(ii) Calculate the period of the circular motion of the sphere.

period = s [2]

[Total: 12]

Q10 (a) Define the radian.

..... [1]

(b) The minute hand of a clock revolves at constant angular speed around the face of the clock, completing one revolution every hour. A small piece of modelling clay is attached to the hand with its centre of gravity at a distance L from the fixed end of the hand, as shown in Fig. 1.1.

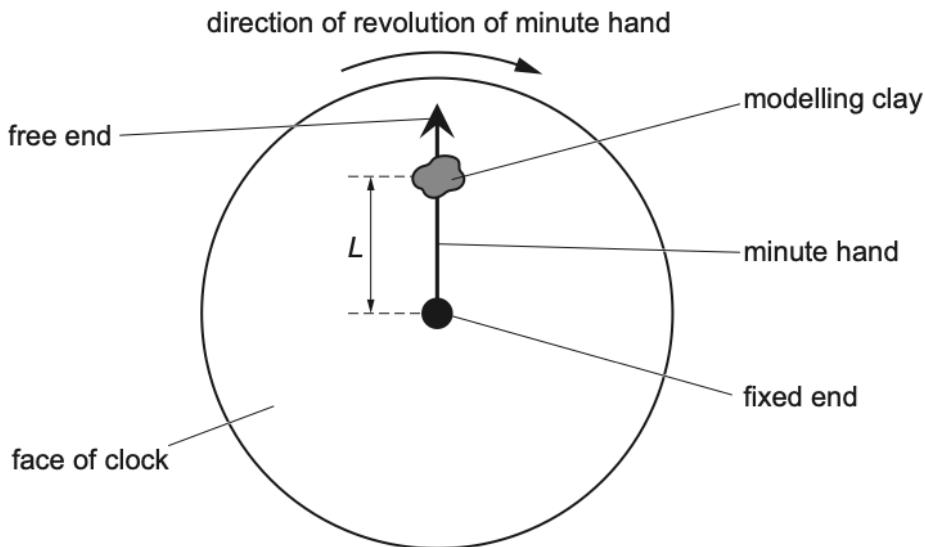


Fig. 1.1

Calculate the angular speed ω of the minute hand.

$$\omega = \dots \text{ rad s}^{-1} [2]$$

(c) During a time interval of 1400 s, the centre of gravity of the piece of modelling clay in Fig. 1.1 moves through a total distance of 0.44 m.

(i) Calculate the angle through which the minute hand moves in this time interval.

$$\text{angle} = \dots \text{ rad} [1]$$

(ii) Determine distance L .

$$L = \dots \text{m} [2]$$

(iii) Calculate the magnitude of the centripetal acceleration of the piece of modelling clay.

$$\text{centripetal acceleration} = \dots \text{ms}^{-2} [2]$$

(d) Use your answer in (c)(iii) to explain why the variation with time of the magnitude of the force exerted by the minute hand on the piece of modelling clay is negligible as the minute hand undergoes one full revolution.

.....
.....
..... [2]

[Total: 10]

Q11 A metal wheel consists of an axle A, eight spokes and a rim, as shown in Fig. 1.1.

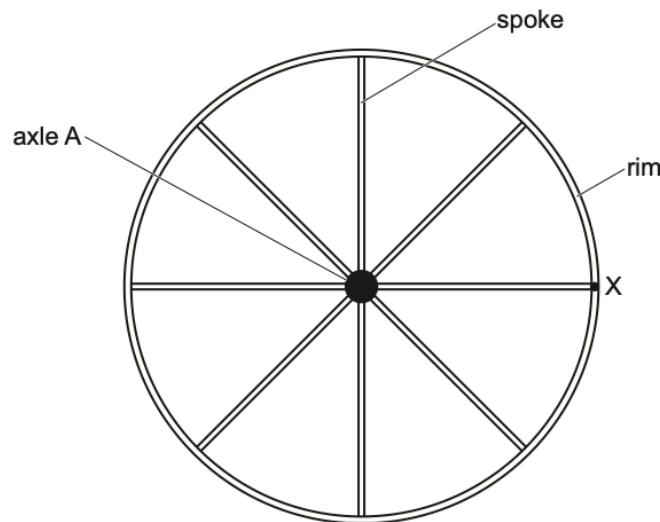


Fig. 1.1

Point X is on the rim at the end of one of the spokes.

The rim has a radius of 0.85 m.

The wheel is rotating clockwise with an angular speed of 140 rad s^{-1} .

(a) For point X, determine:

(i) the speed

$$\text{speed} = \dots \text{ ms}^{-1} \quad [2]$$

(ii) the centripetal acceleration.

$$\text{acceleration} = \dots \text{ ms}^{-2} \quad [2]$$

Q12 A steel ball is placed on the inside surface of a hollow circular cone. The ball moves in a horizontal circle at constant speed, as shown in Fig. 1.1.

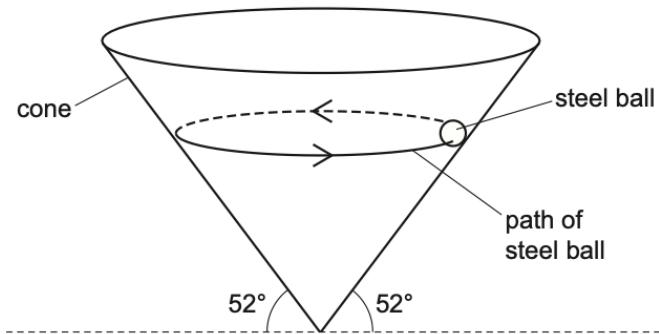


Fig. 1.1

The angle of the side of the cone to the horizontal is 52° . There is no friction between the ball and the cone.

(a) Fig. 1.2 shows a cross-section through the cone and the steel ball.

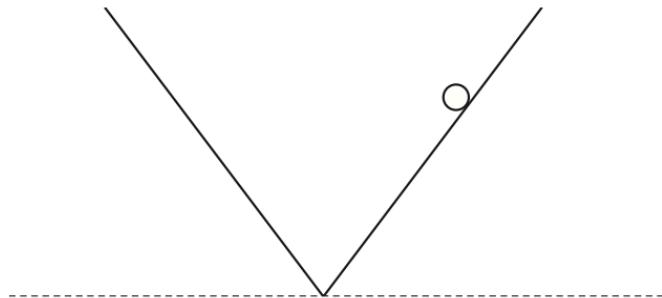


Fig. 1.2

On Fig. 1.2, draw labelled arrows to show the **two** forces acting on the ball. [1]

(b) Describe how the forces acting on the ball cause its acceleration to be centripetal.

.....
.....
.....
.....

[2]

(c) The ball moves in a circle of radius 0.15 m.

Show that the speed of the ball is 1.4 m s^{-1} .

[3]

(d) Calculate the angular speed ω of the ball.

$$\omega = \dots \text{ rad s}^{-1} \quad [2]$$

(e) The speed of the ball is increased.

Explain why the radius of the circular path of the ball increases.

.....
.....
.....

[1]

[Total: 9]

Q13 (a) Define the radian.

MJ25/42/Q1

.....
..... [1]

(b) The rear wheel and the pedals of a bicycle are connected by a chain that passes around two cogs (toothed wheels), as shown in Fig. 1.1.

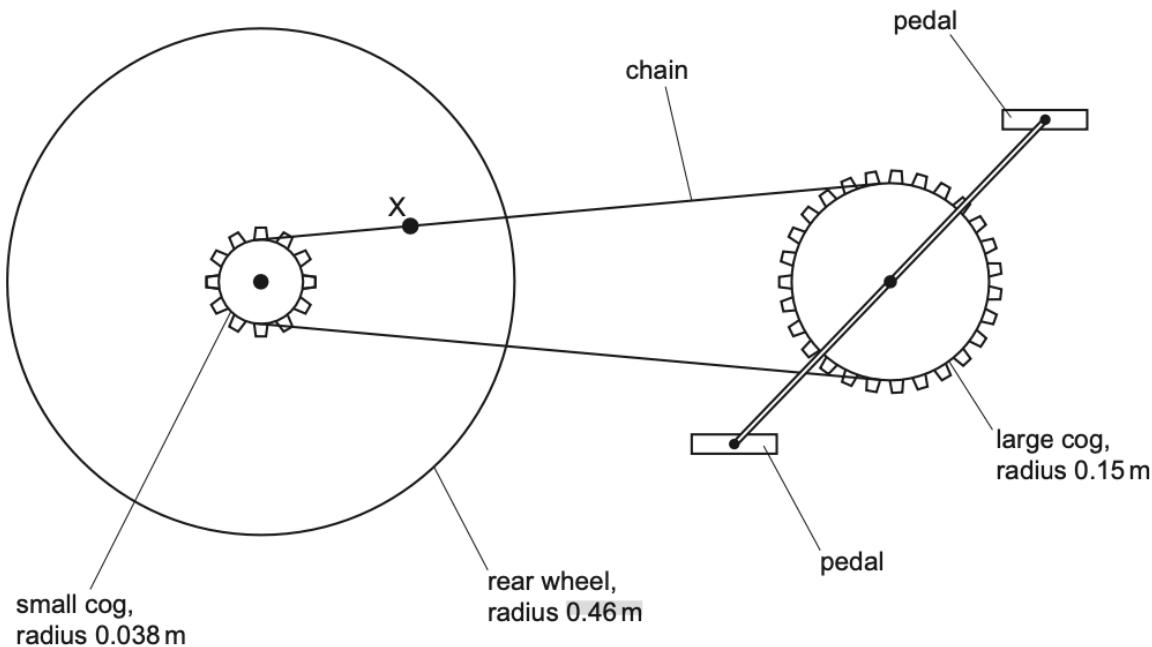


Fig. 1.1 (not to scale)

The small cog has a radius of 0.038 m and is fixed to the rear wheel so that it rotates with it. The large cog has a radius of 0.15 m and is fixed to the pedals so that it rotates with them. The rear wheel has a radius of 0.46 m.

The bicycle is being pedalled so that it moves in a straight line at a constant speed of 17 m s^{-1} .

(i) Calculate the angular speed of the rear wheel.

angular speed = rad s^{-1} [2]

(ii) Calculate the period of rotation of the small cog.

period = s [2]

(iii) Show that the distance moved by point X on the chain during one full rotation of the small cog is 0.24 m.

[1]

(iv) Use the information in (b)(iii) to determine the angle through which the large cog rotates during one full rotation of the small cog.

angle = rad [2]

(c) The chain of the bicycle in (b) is moved onto a smaller cog fixed to the rear wheel. The speed of the bicycle does not change.

Explain, without calculation, the effect of this change on the angular speed of the pedals.

.....
.....
..... [2]

[Total: 10]

Mark scheme

Q1.

Question	Answer	Marks
1(a)	constant speed or constant magnitude of velocity acceleration (always) perpendicular to velocity	B1 B1
1(b)(i)	$F = mv^2/r$ or $v = r\omega$ and $F = m r \omega^2$	C1
	$F = 790 \times 94^2 / 318$ $= 22000 \text{ N}$	A1
1(b)(ii)	centripetal acceleration: same maximum speed: greater time taken for one lap of the track: greater	B1 B1 B1

Q2.

Question	Answer	Marks
1(a)	acceleration perpendicular to velocity	B1
1(b)(i)	decreases	B1
1(b)(ii)	(acceleration of) 9.8 ms^{-2} is caused by weight of car or centripetal force must be greater than weight of car	B1
	(acceleration $> 9.8 \text{ ms}^{-2}$) requires contact <u>force</u> from track or (centripetal force $>$ weight) requires contact <u>force</u> from track	B1
1(c)	$\frac{1}{2}mv_y^2 = \frac{1}{2}mv_x^2 - mgh$	C1
	$a = v^2/r$	C1
	$v_y^2 = 3.8^2 - 2 \times 9.81 \times 0.62$ so $v_y = 1.5 \text{ ms}^{-1}$	A1
	$a = 1.5^2 / 0.31 = 7.3 \text{ ms}^{-2}$ (which is less than 9.8 ms^{-2}) so no	
	or	
	$v_y = \sqrt{9.81 \times 0.31} = 1.74 \text{ m s}^{-1}$ so $v_x^2 = 1.74^2 + 2 \times 9.81 \times 0.62$	(A1)
	$v_x = 3.9 \text{ m s}^{-1}$ (which is greater than 3.8 m s^{-1}) so no	
1(d)	acceleration is independent of mass so makes no difference or mass cancels in the equation so makes no difference	B1

Q3. (a) angle subtended at the centre of a circle
by an arc equal in length to the radius

B1
B1 [2]

(b) (i) arc = distance \times angle
diameter = $3.8 \times 10^5 \times 9.7 \times 10^{-6}$
= 3.7 km

C1
A1 [2]

(ii) Mars is (much) further from Earth/away (answer must be comparative)
angle (at telescope is much) smaller

B1
B1 [2]

Q4. (a) (i) $F = R \cos \theta$ $W = R \sin \theta$ dividing, $W = F \tan \theta$ (max. 1 if derivation to final line not shown)	M1 M1 A0 [2]
(ii) <u>provides</u> the centripetal force	B1 [1]
(b) either $F = mv^2/r$ and $W = mg$ or $v^2 = rg/\tan \theta$ $v^2 = (14 \times 10^{-2} \times 9.8)/\tan 28^\circ$ $= 2.58$ $v = 1.6 \text{ ms}^{-1}$	C1 C1 A1 [3]
Q5. (a) angle (subtended) <u>at centre</u> of circle (by) arc equal in length to radius	B1 B1 [2]
(b) (i) point S shown below C	B1 [1]
(ii) (max) force / tension = weight + centripetal force centripetal force = $mr\omega^2$ $15 = 3.0/9.8 \times 0.85 \times \omega^2$ $\omega = 7.6 \text{ rad s}^{-1}$	C1 C1 C1 A1 [4]
Q6. (a) (i) $(\theta =) \omega t$ (allow any subject if all terms given)	B1 [1]
(ii) $(SQ =) r \sin \omega t$ (allow any subject if all terms given)	B1 [1]
(b) this is the solution of the equation $a = -\omega^2 x$ $a = -\omega^2 x$ is the (defining) equation of s.h.m.	M1 A1 [2]
(c) (i) $f = \omega / 2\pi$ $= 4.7 / 2\pi$ $= 0.75 \text{ Hz}$	C1 A1 [2]
(ii) $v = r\omega$ (r must be identified) $= 4.7 \times 12$ $= 56 \text{ cm s}^{-1}$	C1 A1 [2]

Q7. (a) (i)	angle (subtended) at centre of circle by an arc equal in length to the radius (of the circle)	B1 B1	[2]
	angle swept out per unit time / rate of change of angle by the string	M1 A1	[2]
(b)	friction provides / equals the centripetal force $0.72 W = md\omega^2$ $0.72 mg = m \times 0.35\omega^2$ $\omega = 4.49 \text{ rad s}^{-1}$ $n = (\omega/2\pi) \times 60$ $= 43 \text{ min}^{-1}$ (allow 42)	B1 C1 C1 B1 A1	[5]
(c)	either centripetal force increases as r increases or centripetal force larger at edge so flies off at edge first ($F = mr\omega^2$ so edge first – treat as special case and allow one mark)	M1 A1	[2]
Q8. (a) (i)	angle subtended at centre of circle	B1	
	arc equal in length to the radius	B1	[2]
(ii)	arc = $r\theta$ and for one revolution, arc = $2\pi r$	M1	
	so, $\theta = 2\pi r/r = 2\pi$	A0	[1]
(b) (i)	either weight provides>equals the centripetal force or acceleration of free fall is centripetal acceleration	B1	
	$9.8 = 0.13 \times \omega^2$	M1	
	$\omega = 8.7 \text{ rad s}^{-1}$	A0	[2]
(ii)	force in cord = weight + centripetal force (can be an equation)	C1	
	force in cord = $(L - 13) \times 5/1.8$ or force constant = $5.0/1.8$	C1	
	$(L - 13) \times 5/1.8 = 5.0 + 5/9.8 \times L \times 10^{-2} \times 8.7^2$	C1	
	$L = 17.2 \text{ cm}$	A1	[4]
	(constant centripetal force of 5.0 N gives $L = 16.6 \text{ cm}$ allow 2/4)		

Q9. (a)	horizontal force on sphere causes centripetal acceleration	B1
	weight of sphere is (now) equal to vertical component of tension or horizontal and vertical components (of force) (now) combine to give greater tension (in spring)	B1
	greater tension in spring so greater extension of spring	B1
2(b)(i)	$r = 10.8 \times \sin 27^\circ = 4.9 \text{ cm}$	A1
2(b)(ii)	$T \cos \theta = mg$ or $T \cos \theta = W \text{ and } W = mg$	C1
	$T \cos 27^\circ = 0.29 \times 9.81 \text{ leading to } T = 3.2 \text{ N}$	A1
2(b)(iii)	$\Delta T = 3.2 - (0.29 \times 9.81)$	C1
	$k = \Delta T / \Delta x$ $= [3.2 - (0.29 \times 9.81)] / [10.8 - 8.5]$ $= 0.15 \text{ N cm}^{-1}$	A1
	centripetal acceleration $= (T \sin \theta) / m$ $= (3.2 \times \sin 27^\circ) / 0.29$ $= 5.0 \text{ m s}^{-2}$	C1
2(c)(i)	$a = r\omega^2 \text{ and } \omega = 2\pi / T$ or $a = v^2 / r \text{ and } v = 2\pi r / T$	C1
	$T = 2\pi \times \sqrt{(0.049 / 5.0)}$ $= 0.62 \text{ s}$	A1

Q10	a) angle (subtended at centre of circle) when arc length = radius	B1
1(b)	$\omega = 2\pi / T$ $= 2\pi / (1.0 \times 60 \times 60)$ $= 1.7 \times 10^{-3} \text{ rad s}^{-1}$	C1 A1
1(c)(i)	angle $= 1.7 \times 10^{-3} \times 1400$ $= 2.4 \text{ rad}$	A1
1(c)(ii)	$L = \text{arc length} / \text{angle}$ $= 0.44 / 2.4$ or $L = 0.44 \times (3600 / 1400) / 2\pi$ $L = 0.18 \text{ m}$	C1 A1
1(c)(iii)	$a = r\omega^2$ $= 0.18 \times (1.745 \times 10^{-3})^2$ $= 5.5 \times 10^{-7} \text{ m s}^{-2}$	C1 A1
1(d)	centripetal acceleration is negligible compared with acceleration of free fall or numerical comparison establishing answer to (c)(iii) $\ll 9.81$ resultant force is negligible compared with weight (of modelling clay) (so variation is negligible) or force exerted by minute hand (approximately) equal (and opposite) to weight of modelling clay	B1 B1

Q11	a)(i)	$v = r\omega$	C1
		$= 0.85 \times 140$	A1
		$= 120 \text{ m s}^{-1}$	
	1(a)(ii)	$a = r\omega^2$ or $a = v^2 / r$	C1
		$a = 0.85 \times 140^2$ or $120^2 / 0.85$	A1
		$= 1.7 \times 10^4 \text{ m s}^{-2}$	

Q12	a)	arrow vertically downwards labelled 'weight' and arrow perpendicular to cone, inwards and upwards, labelled 'normal contact force'	B1
1(b)		vertical component of contact force = weight (so no resultant force vertically)	B1
		horizontal component of contact force is resultant force towards centre (of circle)	B1
1(c)		$a = v^2 / r$	C1
		$a = g \tan 52^\circ$	C1
		$9.81 \times \tan 52^\circ = v^2 / 0.15$ leading to $v = 1.4 \text{ m s}^{-1}$	A1
1(d)		$v = r\omega$ or $a = r\omega^2$	C1
		$\omega = 1.4 / 0.15$ or $\sqrt{(9.81 \times \tan 52^\circ / 0.15)}$ $= 9.3 \text{ rad s}^{-1}$	A1
1(e)		same resultant force / same acceleration so v^2 is proportional to r (so if speed increases radius must also increase)	A1

Q13	a) angle subtended (at centre of a circle) when arc length = radius	B1	Ignore 'ratio arc length / radius'.
1(b)(i)	$v = r\omega$ $\omega = 17 / 0.46$ $= 37 \text{ rad s}^{-1}$	C1	
1(b)(ii)	$T = 2\pi r / v$ or $T = 2\pi / \omega$ $= 2\pi \times 0.46 / 17$ or $2\pi / 37$ $= 0.17 \text{ s}$	C1	Possible ECF from (b)(i). A1 Correct to at least 2 significant figures. AFC. (0.170)
1(b)(iii)	distance = $2\pi r$ $= 2\pi \times 0.038 = 0.24 \text{ m}$	A1	Full substitution and answer needed.
1(b)(iv)	angle = arc length / radius $= 0.24 / 0.15$ $= 1.6 \text{ rad}$	C1	No ECF from (b)(iii). A1 Correct to at least 2 significant figures. AFC. (1.60)
1(c)	point X moves through a smaller distance in the same time angular speed of pedals decreases	B1	
		B1	