

DYNAMICS

AS LEVEL WORKSHEET

MJ25/22/Q4

- 1 A small ball is dropped from rest from height h_1 above the ground and falls vertically downwards. The ball collides with the ground and bounces back vertically upwards, reaching a maximum height h_2 . Fig. 4.1 shows the ball just before and just after hitting the ground.

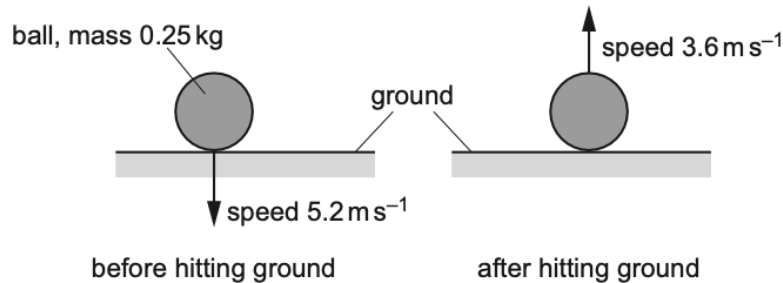


Fig. 4.1

The ball has mass 0.25 kg and is in contact with the ground for a time of 0.18 s . Just before the ball hits the ground, it has speed 5.2 m s^{-1} . Just after it leaves the ground, it has speed 3.6 m s^{-1} . Air resistance acting on the ball is negligible.

- (a) State and explain whether the collision is elastic or inelastic.

.....

 [1]

- (b) (i) Calculate the change in momentum of the ball during the collision with the ground.

change in momentum = kg m s^{-1} [2]

- (ii) Determine the average force on the ball during the collision with the ground.

force = N [2]

- (c) Calculate the ratio $\frac{h_2}{h_1}$.

ratio = [3]

[Total: 8]

MJ25/23/Q4

- 2 (a) State the principle of conservation of momentum.

.....

 [2]

- (b) An object A of mass 4.0 kg travels at a velocity of 6.0 ms^{-1} to the right on a horizontal frictionless surface. It moves towards a second object B of mass 2.0 kg that is moving at a velocity of 3.0 ms^{-1} in the same direction as A, as shown in Fig. 4.1.

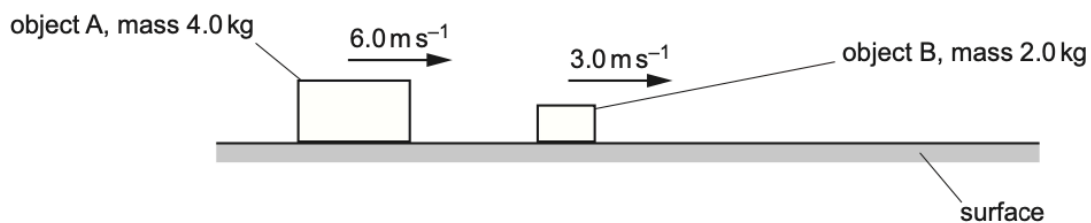


Fig. 4.1

Object A collides with object B. The two objects join and move off together with velocity v .

Calculate:

- (i) velocity v

$v = \dots \text{ ms}^{-1}$ [2]

- (ii) the percentage of the total initial kinetic energy of the two objects that is transferred to other forms of energy during the collision.

percentage = % [2]

[Total: 6]

MJ25/24/Q2

- 3** An object of constant mass moves in a straight line. The variation with time t of the momentum p of the object is shown in Fig. 2.1.

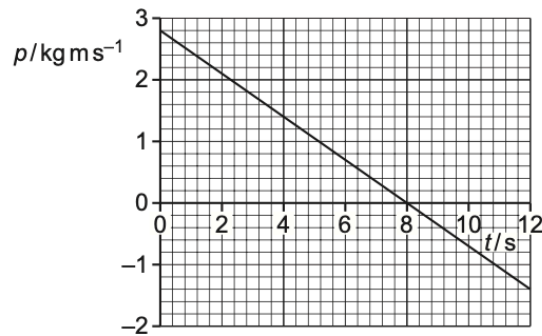


Fig. 2.1

- (a) Define momentum.

.....
 [1]

- (b) Calculate the change in momentum of the object from time $t = 0$ to $t = 12$ s.

change in momentum = kg ms⁻¹ [1]

- (c) Calculate the magnitude of the resultant force acting on the object.

force = N [2]

- (d) Describe the variation of the speed of the object from time $t = 0$ to $t = 8.0$ s.

.....
..... [1]

- (e) By reference to Fig. 2.1, explain why the resultant force acting on the object during the first 8.0 s of its motion cannot be due to air resistance.

.....
.....
..... [2]

- (f) At time $t = 0$ the displacement d of the object is zero.

On Fig. 2.2, sketch the variation of d with time t from $t = 0$ to $t = 12$ s.

Numerical values of d are not required.

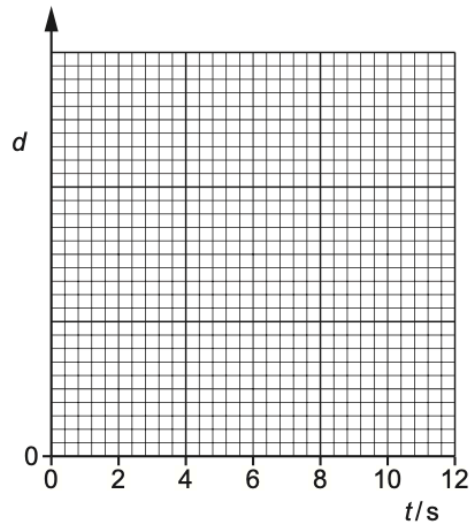


Fig. 2.2

[3]

[Total: 10]

- 4 (a) A truck R of mass 9400 kg moves with constant acceleration in a straight line down a slope, as illustrated in Fig. 3.1.

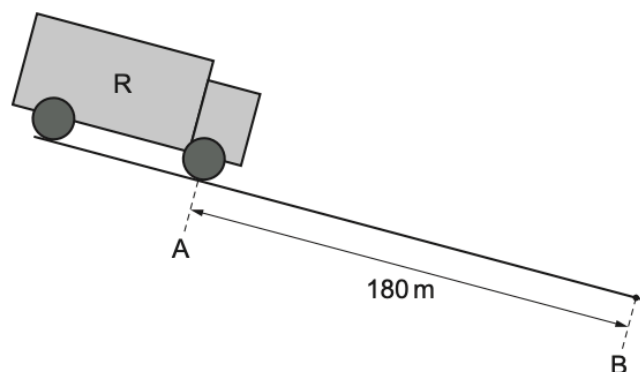


Fig. 3.1

At point A the speed of the truck is 13 ms^{-1} and at point B the speed of the truck is 22 ms^{-1} . A and B are a distance of 180 m apart.

- (i) Calculate the acceleration of the truck between A and B.

acceleration = ms^{-2} [2]

- (ii) Determine the gain in kinetic energy of the truck between A and B.

gain in kinetic energy = J [3]

- (b) A short time after passing point B truck R moves in a straight line on horizontal ground. The driver of the truck applies the brakes. Fig. 3.2 shows the variation with time of the momentum of the truck.

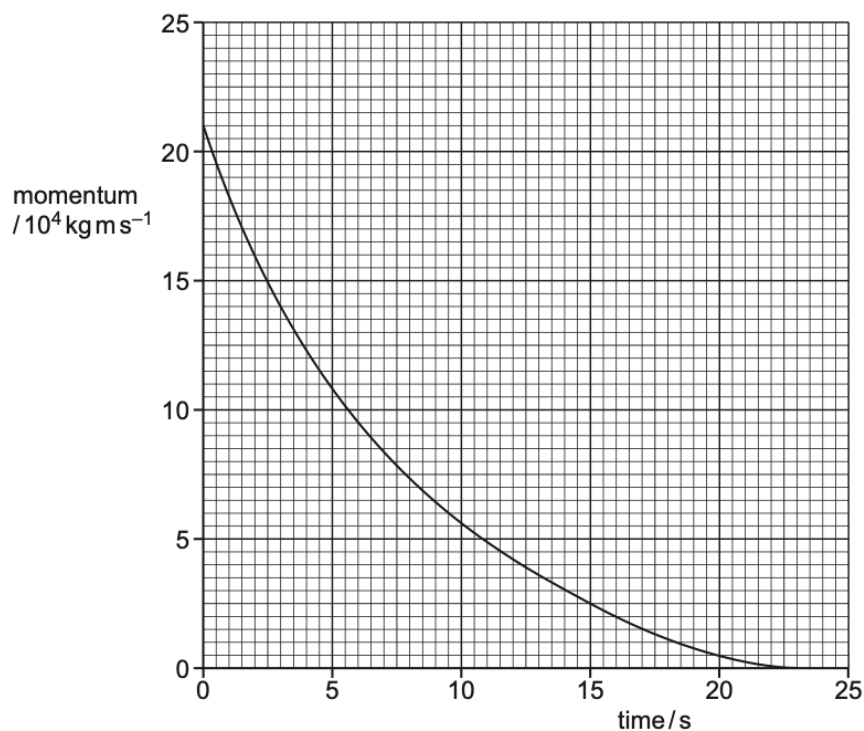


Fig. 3.2

- (i) Define force.

.....
 [1]

- (ii) Show that the average resultant force F acting on truck R between time $t = 0$ and $t = 15 \text{ s}$ is $-1.2 \times 10^4 \text{ N}$.

[1]

- (iii) An identical truck S has the same initial momentum as truck R. Truck S experiences a constant force equal to the force F in (b)(ii).

State and explain whether truck S will take more, less or the same amount of time to come to rest as truck R.

.....

.....

.....

.....

.....

.....

..... [3]

[Total: 10]

5 (a) Define linear momentum.

ON24/21/Q2

.....
..... [1]

(b) A car of mass 1800 kg is moving in a straight line. Fig. 2.1 shows the variation with time t of the momentum p of the car.

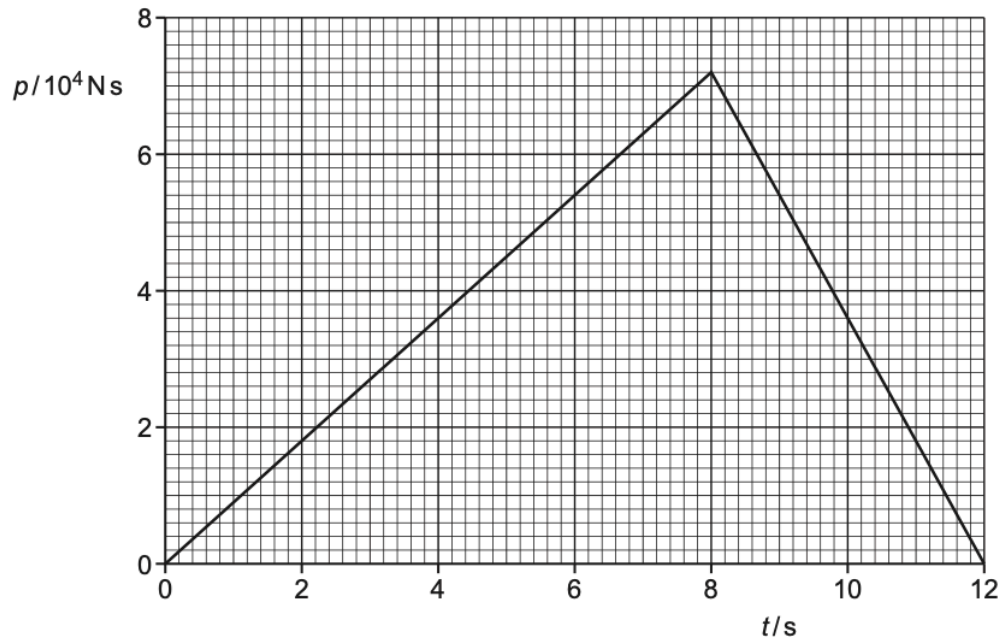


Fig. 2.1

(i) Calculate the maximum speed reached by the car.

maximum speed = ms^{-1} [1]

(ii) Calculate the maximum kinetic energy of the car.

maximum kinetic energy = J [2]

(iii) Show that the acceleration of the car at time $t = 4.0\text{ s}$ is 5.0 ms^{-2} .

[2]

(iv) Determine the distance travelled by the car between times $t = 0$ and $t = 12.0\text{ s}$.

distance = m [2]

(c) On Fig. 2.2, sketch the variation with time t of the acceleration a of the car in (b) from $t = 0$ to $t = 12.0\text{ s}$.

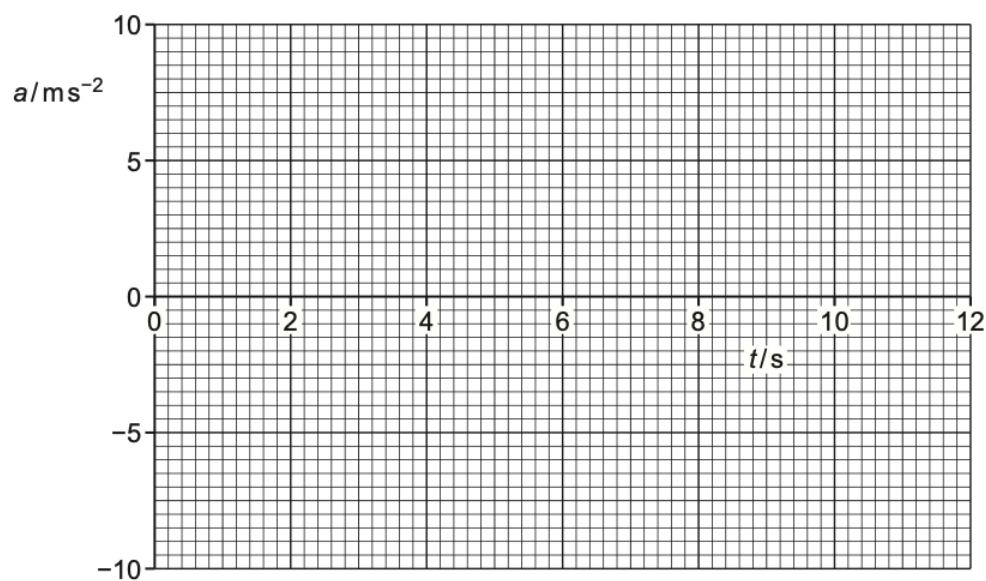


Fig. 2.2

[3]

[Total: 11]

- 6 (a) State the principle of conservation of momentum.

.....

 [2]

- (b) A ball X has mass 240 g and moves in a straight line on a horizontal frictionless surface with an initial speed of 16 m s^{-1} . The ball collides with a stationary ball Y that has mass 480 g. After the collision, ball X is stationary, as shown in Fig. 2.1.

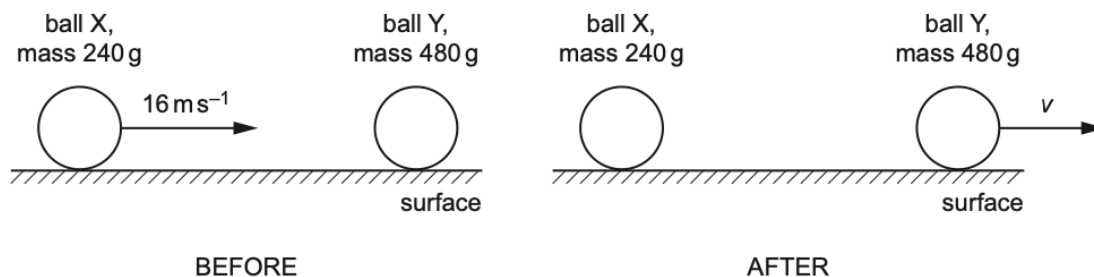


Fig. 2.1

- (i) Show that the speed v of ball Y after the collision is 8.0 m s^{-1} .

[1]

- (ii) Calculate the change in the total kinetic energy ΔE_K of the balls due to the collision.

$\Delta E_K = \dots\dots\dots \text{ J [3]}$

- (c) The collision in (b) lasts for a time of 2.0 ms. Assume that the contact force between the balls is constant during this time.
- (i) Determine the magnitude and direction of the force exerted on ball X by ball Y during the collision.

magnitude = N

direction [3]

- (ii) Compare the magnitude and direction of the force exerted on ball Y by ball X during the collision with the answers in (c)(i). No further calculations are required.

.....

 [2]

[Total: 11]

- 7 (a) Define momentum.

.....
 [1]

- (b) A child stands on a scooter on horizontal ground. The combined mass of the child and the scooter is 16 kg.
 The child starts from rest and pushes once on the ground with her foot which causes her to accelerate. The push lasts for a time of 1.1 s. The speed of the child and the scooter after the push is 0.60 m s^{-1} .

Determine the average resultant force acting horizontally on the child and the scooter during the push.

average force = N [2]

- (c) Later, the child in (b) travels down a slope at a constant angle to the horizontal, as shown in Fig. 2.1.

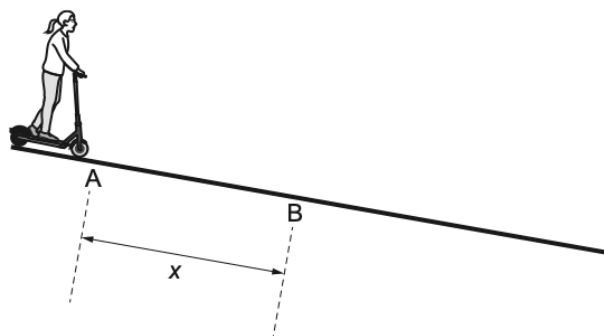


Fig. 2.1 (not to scale)

At point A her speed is 0.60 m s^{-1} . She has a constant acceleration of 0.85 m s^{-2} parallel to the slope. After a time of 3.7 s, she reaches point B.

Calculate the distance x travelled by the child along the slope from A to B.

$x = \dots\dots\dots \text{ m}$ [2]

- (d) At point B, the child in (c) applies the brake with a constant force to maintain a constant velocity. Point C is 18 m from point B, as shown in Fig. 2.2.

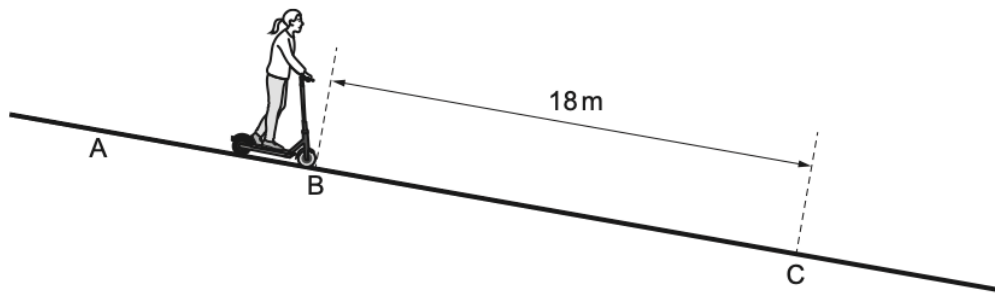


Fig. 2.2 (not to scale)

The work done by the braking force between B and C is 250 J.

- (i) Determine the magnitude of the braking force.

force = N [2]

- (ii) On Fig. 2.3, sketch the variation of the kinetic energy of the child and scooter with distance travelled from point A to point C. Numerical values for kinetic energy are not required.

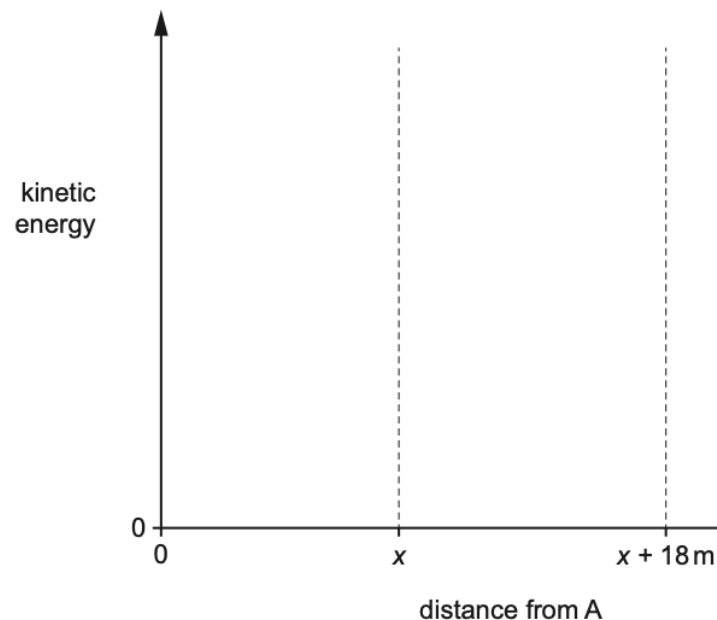


Fig. 2.3

[3]

[Total: 10]

- 8 (a) State the principle of conservation of momentum.

.....

 [2]

- (b) An object of mass $2m$ is travelling at a speed of 5.0 ms^{-1} in a straight line. It collides with an object of mass $3m$ which is initially stationary, as shown in Fig. 3.1.

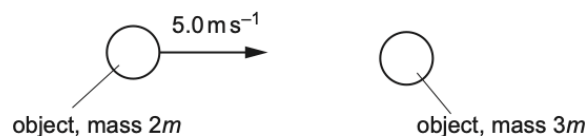
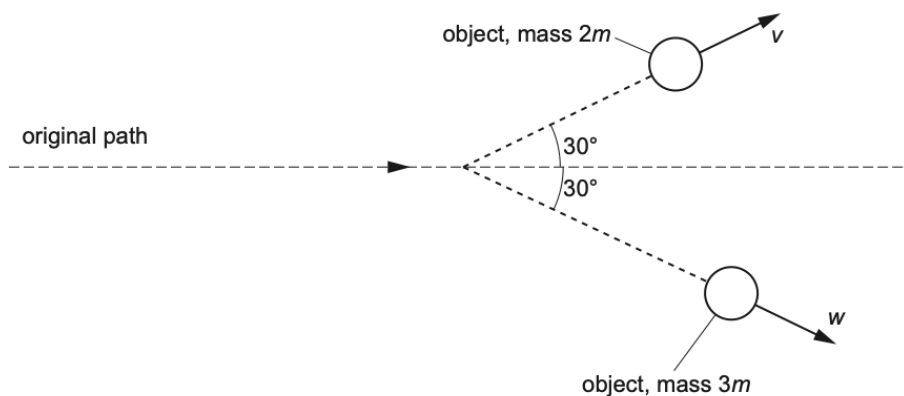


Fig. 3.1

After the collision, the object of mass $2m$ moves with velocity v at an angle of 30° to its original direction of motion.

The object of mass $3m$ moves with velocity w also at an angle of 30° , as shown in Fig. 3.2.



By considering the conservation of momentum in two dimensions, calculate the magnitudes of v and w .

$v = \dots\dots\dots \text{ms}^{-1}$

$w = \dots\dots\dots \text{ms}^{-1}$
 [4]

- (c) An object of mass 4.2 kg is travelling in a straight line at a speed of 6.0 ms^{-1} . The object is brought to rest in a distance of 0.050 m by a constant force.

Calculate the magnitude of this force.

force = N [3]

[Total: 9]

ON23/21/Q3

- 9 A trolley A moves along a horizontal surface at a constant velocity towards another trolley B which is moving at a lower constant speed in the same direction. Fig. 3.1 shows the trolleys at time $t = 0$.

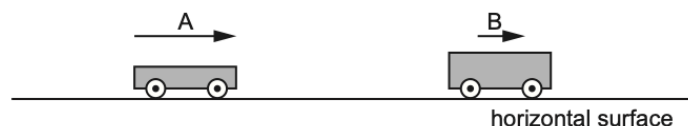


Fig. 3.1

Table 3.1 shows data for the trolleys.

Table 3.1

trolley	mass / kg	initial speed / ms^{-1}
A	0.25	0.48
B	0.75	0.12

The two trolleys collide elastically and then separate. Resistive forces are negligible.

Fig. 3.2 shows the variation with time t of the velocity v for trolley B.

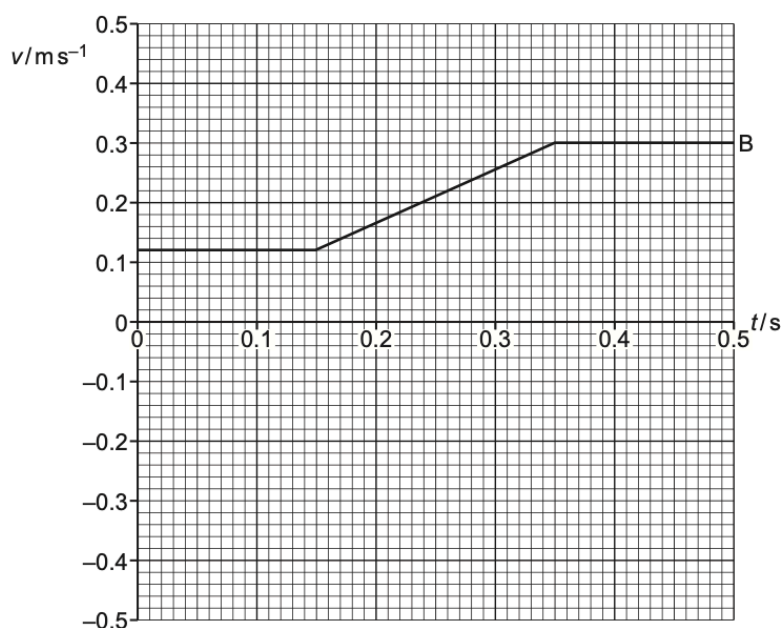


Fig. 3.2

- (a) State what is represented by the area under a velocity–time graph.

..... [1]

- (b) Use Table 3.1 and Fig. 3.2 to determine:

- (i) the acceleration of trolley B during the collision

acceleration of B = ms^{-2} [2]

- (ii) the magnitude and direction of the final velocity of trolley A.

magnitude = ms^{-1}

direction [3]

- (c) On Fig. 3.2, sketch the variation of the velocity of trolley A with time t from $t = 0$ to $t = 0.50$ s.

[3]

[Total: 9]

- 10 (a) A ball Y moves along a horizontal frictionless surface and collides with a ball Z, as illustrated in the views from above in Fig. 4.1 and Fig. 4.2.

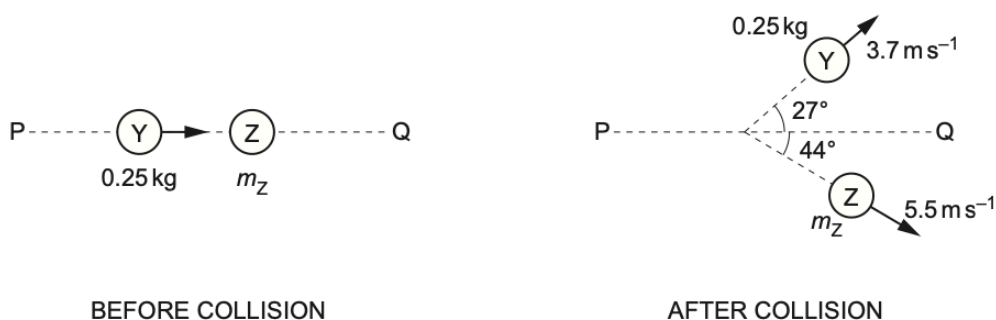


Fig. 4.1 (not to scale)

Fig. 4.2 (not to scale)

Ball Y has a mass of 0.25 kg and initially moves along a line PQ.
Ball Z has a mass m_Z and is initially stationary.

After the collision, ball Y has a final velocity of 3.7 m s^{-1} at an angle of 27° to line PQ and ball Z has a final velocity of 5.5 m s^{-1} at an angle of 44° to line PQ.

- (i) Calculate the component of the final momentum of ball Y in the direction perpendicular to line PQ.

component of momentum = Ns [2]

- (ii) By considering the component of the final momentum of each ball in the direction perpendicular to line PQ, calculate m_Z .

$m_Z = \dots \text{ kg}$ [1]

- (iii) During the collision, the average force exerted on Y by Z is F_Y and the average force exerted on Z by Y is F_Z .

Compare the magnitudes and directions of F_Y and F_Z . Numerical values are not required.

magnitudes:

directions:

[2]

- (b) Two blocks, A and B, move directly towards each other along a horizontal frictionless surface, as shown in the view from above in Fig. 4.3.



Fig. 4.3

The blocks collide perfectly elastically. Before the collision, block A has a speed of 4 m s^{-1} and block B has a speed of 6 m s^{-1} . After the collision, block B moves back along its original path with a speed of 2 m s^{-1} .

Calculate the speed of block A after the collision.

speed = m s^{-1} [1]

[Total: 6]

MJ23/21/Q3

- 11 A block is pulled in a straight line along a rough horizontal surface by a varying force X , as shown in Fig. 3.1.

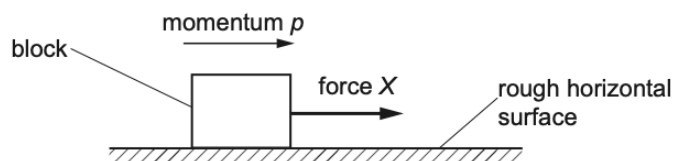


Fig. 3.1

Air resistance is negligible. Assume that the frictional force exerted on the block by the surface is constant and has magnitude 2.0 N .

The variation with time t of the momentum p of the block is shown in Fig. 3.2.

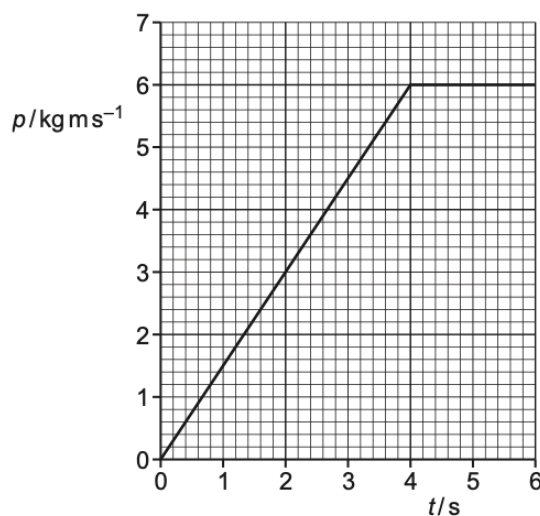


Fig. 3.2

- (a) State Newton's second law of motion.

.....
 [1]

- (b) Use Fig. 3.2 to determine, for the block at time $t = 2.0$ s, the magnitude of:

- (i) the resultant force on the block

resultant force = N [1]

- (ii) the force X .

$X =$ N [1]

- (c) On Fig. 3.3, sketch a graph to show the variation of force X with time t from $t = 0$ to $t = 6.0$ s.

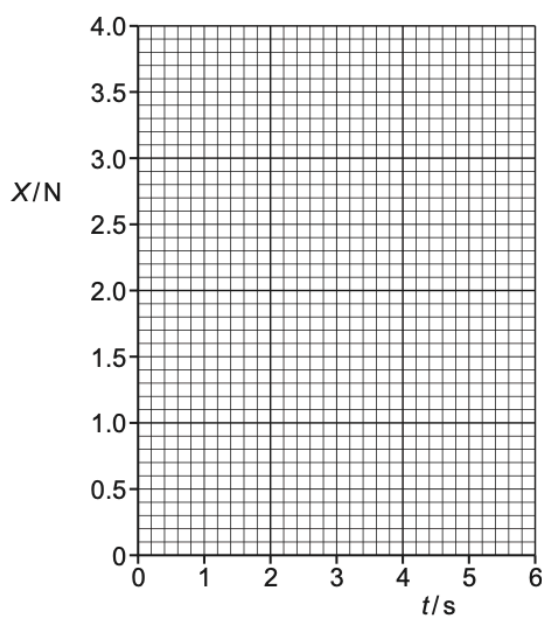


Fig. 3.3

[3]

[Total: 6]

- 12 A block is pulled by a force X in a straight line along a rough horizontal surface, as shown in Fig. 3.1.

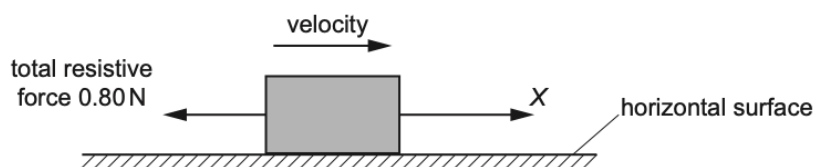


Fig. 3.1

Assume that the total resistive force opposing the motion of the block is 0.80 N at all speeds of the block.

The variation with time t of the magnitude of the force X is shown in Fig. 3.2.

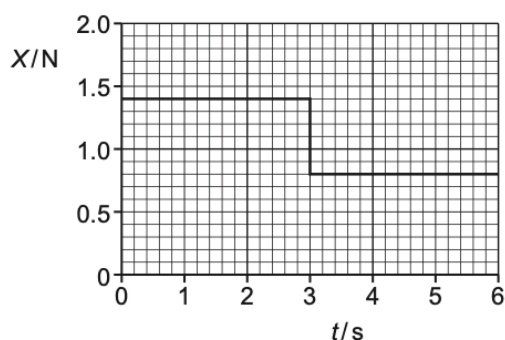


Fig. 3.2

- (a) (i) Define force.

.....
 [1]

- (ii) Determine the change in momentum of the block from time $t = 0$ to time $t = 3.0$ s.

change in momentum = kg ms^{-1} [2]

- (b) (i) Describe and explain the motion of the block between time $t = 3.0$ s and time $t = 6.0$ s.

.....

 [2]

- (ii) Force X produces a total power of 2.0 W when moving the block between time $t = 3.0\text{ s}$ and time $t = 6.0\text{ s}$.

Calculate the distance moved by the block during this time interval.

distance = m [3]

- (c) The block is at rest at time $t = 0$.

On Fig. 3.3, sketch a graph to show the variation of the momentum of the block with time t from $t = 0$ to $t = 6.0\text{ s}$.

Numerical values of momentum are not required.

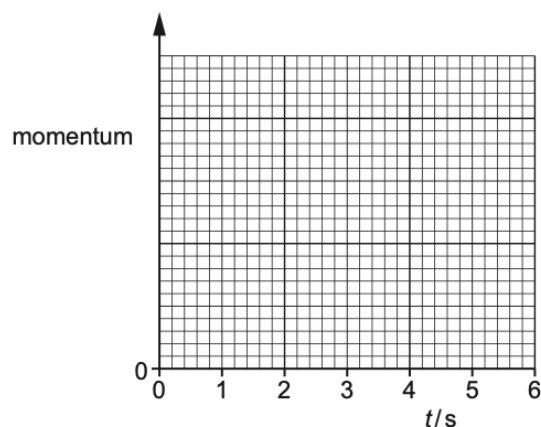


Fig. 3.3

[2]

[Total: 10]

- 13 (a) State the principle of conservation of momentum.

.....

 [2]

- (b) A firework is initially stationary. It explodes into three fragments A, B and C that move in a horizontal plane, as shown in the view from above in Fig. 3.1.

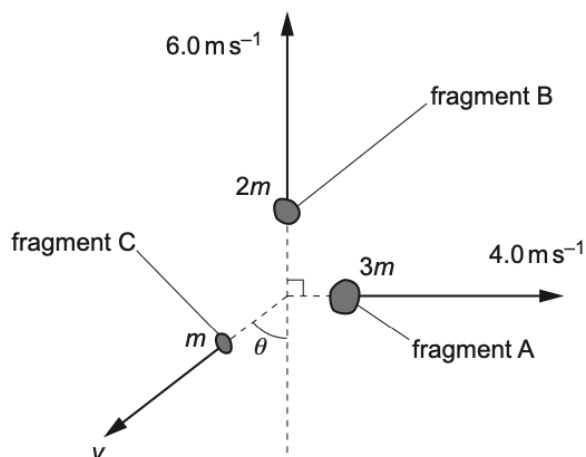


Fig. 3.1

Fragment A has a mass of $3m$ and moves away from the explosion at a speed of 4.0 ms^{-1} .

Fragment B has a mass of $2m$ and moves away from the explosion at a speed of 6.0 ms^{-1} at right angles to the direction of A.

Fragment C has a mass of m and moves away from the explosion at a speed v and at an angle θ as shown in Fig. 3.1.

Calculate:

- (i) the angle θ

$\theta = \dots\dots\dots^\circ$ [3]

(ii) the speed v .

$$v = \dots\dots\dots \text{ms}^{-1} \quad [2]$$

(c) The firework in (b) contains a chemical that has mass 5.0 g and has chemical energy per unit mass 700 J kg^{-1} . When the firework explodes, all of the chemical energy is transferred to the kinetic energy of fragments A, B and C.

(i) Show that the total chemical energy in the firework is 3.5 J.

[1]

(ii) Calculate the mass m .

$$m = \dots\dots\dots \text{kg} \quad [3]$$

[Total: 11]

March23/22/Q4

14 Two blocks slide directly towards each other along a frictionless horizontal surface, as shown in Fig. 4.1. The blocks collide and then move as shown in Fig. 4.2.

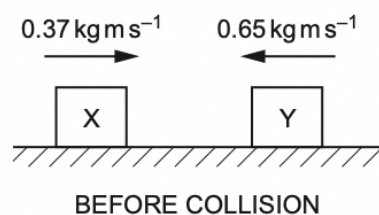


Fig. 4.1

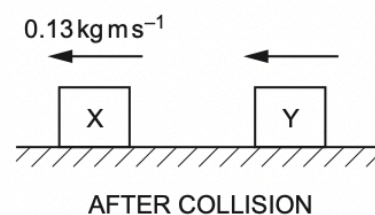


Fig. 4.2

Block X initially moves to the right with a momentum of 0.37 kg m s^{-1} . Block Y initially moves to the left with a momentum of 0.65 kg m s^{-1} . After the blocks collide, block X moves to the left back along its original path with a momentum of 0.13 kg m s^{-1} . Block Y also moves to the left after the collision.

- (a) Block X has an initial kinetic energy of 0.30 J.

Calculate the mass of block X.

mass = kg [3]

- (b) Determine the magnitude of the momentum of block Y after the collision.

momentum = kg ms⁻¹ [1]

..

- (c) Block X exerts an average force of 7.7 N on block Y during the collision.

Calculate the time that the blocks are in contact with each other.

time = s [2]

[Total: 6]

.....

 [2]

- (b) Two balls, X and Y, move along a horizontal frictionless surface, as shown from above in Fig. 4.1.

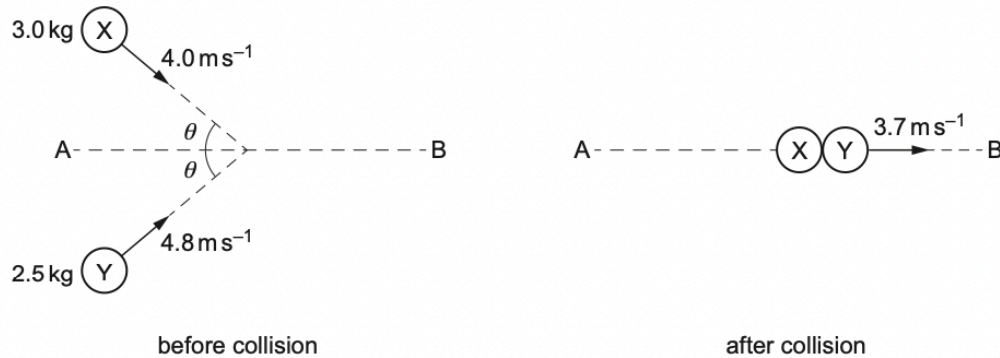


Fig. 4.1 (not to scale)

Fig. 4.2 (not to scale)

Ball X has a mass of 3.0 kg and a velocity of 4.0 m s^{-1} in a direction at angle θ to a line AB. Ball Y has a mass of 2.5 kg and a velocity of 4.8 m s^{-1} in a direction at angle θ to the line AB.

The balls collide and stick together. After colliding, the balls have a velocity of 3.7 m s^{-1} along the line AB on the horizontal surface, as shown in Fig. 4.2.

- (i) By considering the components of the momenta along the line AB, calculate θ .

$\theta = \dots\dots\dots^\circ$ [3]

- (ii) By calculation of kinetic energies, state and explain whether the collision of the balls is inelastic or perfectly elastic.

.....
 [2]

[Total: 7]

- 16 Two blocks slide directly towards each other along a frictionless horizontal surface, as shown in Fig. 4.1. The blocks collide and then move as shown in Fig. 4.2.

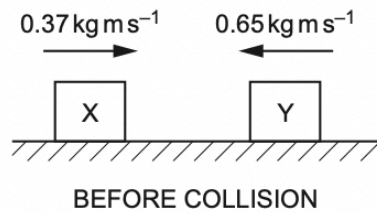


Fig. 4.1

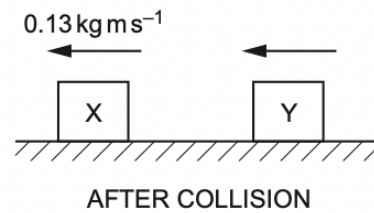


Fig. 4.2

Block X initially moves to the right with a momentum of 0.37 kg m s^{-1} . Block Y initially moves to the left with a momentum of 0.65 kg m s^{-1} . After the blocks collide, block X moves to the left back along its original path with a momentum of 0.13 kg m s^{-1} . Block Y also moves to the left after the collision.

- (a) Block X has an initial kinetic energy of 0.30 J .

Calculate the mass of block X.

mass = kg [3]

- (b) Determine the magnitude of the momentum of block Y after the collision.

momentum = kg m s^{-1} [1]

- (c) Block X exerts an average force of 7.7 N on block Y during the collision.

Calculate the time that the blocks are in contact with each other.

time = s [2]

[Total: 6]

17 (a) Define *momentum*.

.....
 [1]

(b) Two balls X and Y, of equal diameter but different masses 0.24 kg and 0.12 kg respectively, slide towards each other on a frictionless horizontal surface, as shown in Fig. 2.1.

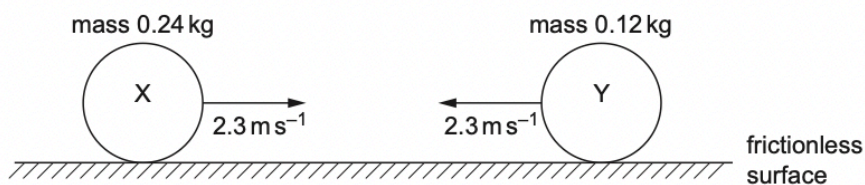


Fig. 2.1

Both balls have initial speed 2.3 ms^{-1} before they collide with each other. Fig. 2.2 shows the variation with time t of the force F_Y exerted on ball Y by ball X during the collision.

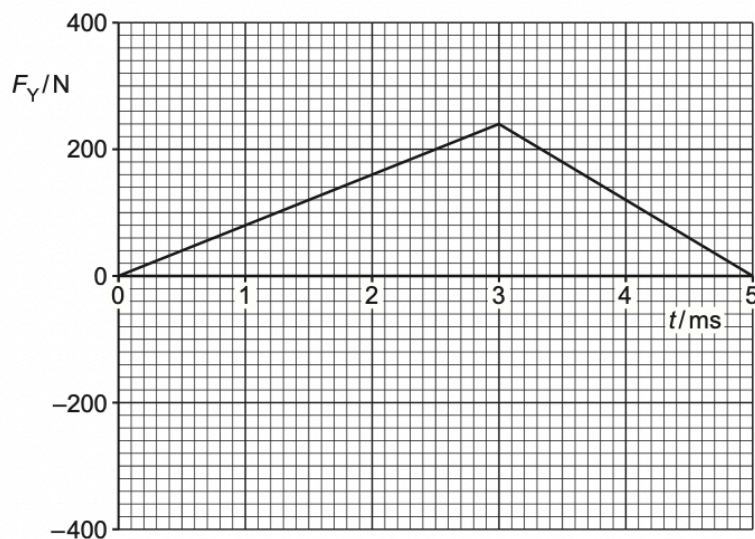


Fig. 2.2

(i) Calculate the kinetic energy of ball X before the collision.

kinetic energy = J [3]

- (ii) The area enclosed by the lines and the time axis in Fig. 2.2 represents the change in momentum of ball Y during the collision.

Determine the magnitude of the change in momentum of ball Y.

change in momentum = Ns [2]

- (iii) Calculate the magnitude of the velocity of ball Y after the collision.

velocity = ms^{-1} [2]

- (c) On Fig. 2.3, sketch the variation with time t of the force F_x exerted on ball X by ball Y during the collision in (b).

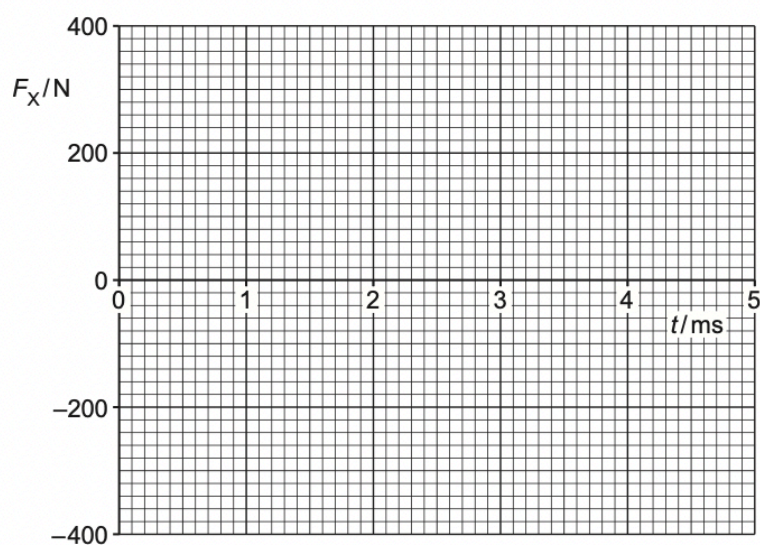


Fig. 2.3

[3]

[Total: 11]

- 18 A pendulum consists of a solid sphere suspended by a string from a fixed point P, as shown in Fig. 3.1.

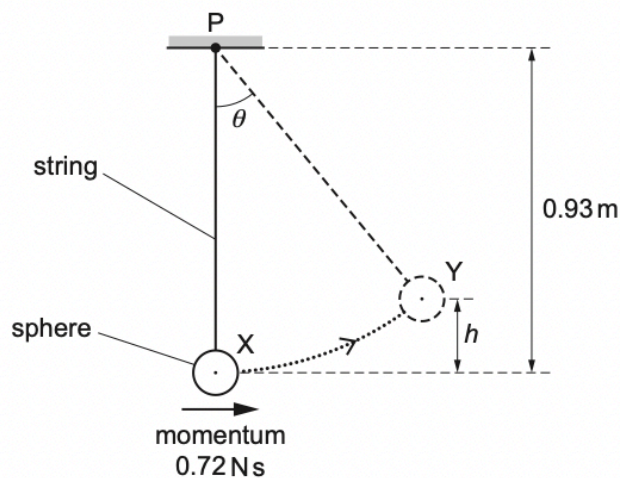


Fig. 3.1 (not to scale)

The sphere swings from side to side. At one instant the sphere is at its lowest position X, where it has kinetic energy 0.86 J and momentum 0.72 N s in a horizontal direction. A short time later the sphere is at position Y, where it is momentarily stationary at a maximum vertical height h above position X.

The string has a fixed length and negligible weight. Air resistance is also negligible.

- (a) On Fig. 3.1, draw a solid line to represent the displacement of the centre of the sphere at position Y from position X. [1]
- (b) Show that the mass of the sphere is 0.30 kg.

[3]

(c) Calculate height h .

$h = \dots\dots\dots$ m [2]

(d) The distance between point P and the centre of the sphere is 0.93 m. When the sphere is at position Y, the string is at an angle θ to the vertical.

Show that θ is 47° .

[1]

(e) For the sphere at position Y, calculate the moment of its weight about point P.

moment = $\dots\dots\dots$ N m [2]

(f) State and explain whether the sphere is in equilibrium when it is stationary at position Y.

$\dots\dots\dots$
 $\dots\dots\dots$ [1]

[Total: 10]

- 19 A ball is thrown vertically downwards to the ground, as illustrated in Fig. 2.1.

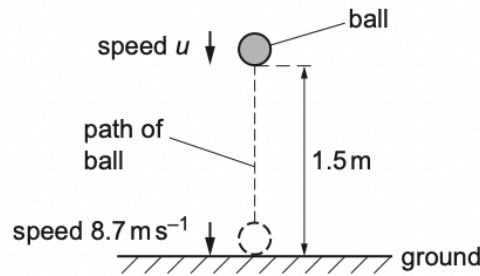


Fig. 2.1

The ball is thrown with speed u from a height of 1.5 m. The ball then hits the ground with speed 8.7 m s^{-1} . Assume that air resistance is negligible.

- (a) Calculate speed u .

$$u = \dots\dots\dots \text{ m s}^{-1} \quad [2]$$

- (b) State how Newton's third law applies to the collision between the ball and the ground.

.....

 [2]

- (c) The ball is in contact with the ground for a time of 0.091 s. The ball rebounds vertically and leaves the ground with speed 5.4 m s^{-1} . The mass of the ball is 0.059 kg.

- (i) Calculate the magnitude of the change in momentum of the ball during the collision.

$$\text{change in momentum} = \dots\dots\dots \text{ N s} \quad [2]$$

- (ii) Determine the magnitude of the average resultant force that acts on the ball during the collision.

average resultant force = N [1]

- (iii) Use your answer in (c)(ii) to calculate the magnitude of the average force exerted by the ground on the ball during the collision.

average force = N [2]

- (d) The ball was thrown downwards at time $t = 0$ and hits the ground at time $t = T$.

On Fig. 2.2, sketch a graph to show the variation of the speed of the ball with time t from $t = 0$ to $t = T$. Numerical values are not required.

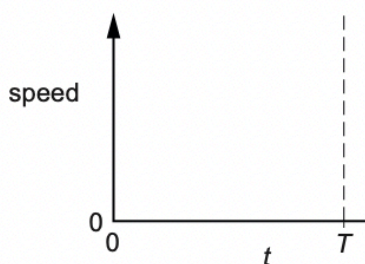


Fig. 2.2

[1]

- (e) In practice, air resistance is not negligible.

State and explain the variation, if any, with time t of the gradient of the graph in (d) when air resistance is not negligible.

.....

.....

.....

..... [2]

[Total: 12]

20 (a) Define *force*.

ON20/21/Q3

.....
..... [1]

- (b) A ball falls vertically downwards towards a horizontal floor and then rebounds along its original path, as illustrated in Fig. 3.1.

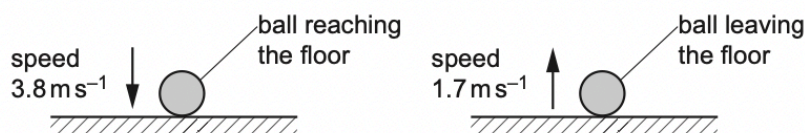


Fig. 3.1

The ball reaches the floor with speed 3.8 ms^{-1} . The ball is then in contact with the floor for a time of 0.081 s before leaving it with speed 1.7 ms^{-1} . The mass of the ball is 0.062 kg .

- (i) Calculate the loss of kinetic energy of the ball during the collision.

loss of kinetic energy = J [2]

- (ii) Determine the magnitude of the change in momentum of the ball during the collision.

change in momentum = Ns [2]

- (iii) Show that the magnitude of the average resultant force acting on the ball during the collision is 4.2 N .

[1]

(iv) Use the information in (iii) to calculate the magnitude of:

1. the average force of the floor on the ball during the collision

average force = N

2. the average force of the ball on the floor during the collision.

average force = N
[2]

[Total: 8]

ON20/22/Q3

- 21 (a) A spring is fixed at one end and is compressed by applying a force to the other end. The variation of the force F acting on the spring with its compression x is shown in Fig. 3.1.

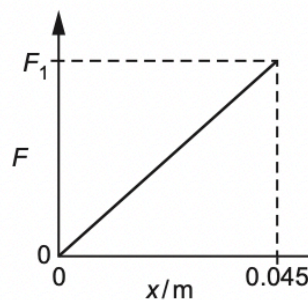


Fig. 3.1

A compression of 0.045 m is produced when a force F_1 acts on the spring. The spring has a spring constant of 800 N m^{-1} .

- (i) Determine F_1 .

$F_1 = \dots\dots\dots$ N [2]

- (ii) Use Fig. 3.1 to show that, for a compression of 0.045 m, the elastic potential energy of the spring is 0.81 J.

[2]

- (b) A child's toy uses the spring in (a) to launch a ball of mass 0.020 kg vertically into the air. The ball is initially held against one end of the spring which has a compression of 0.045 m. The spring is then released to launch the ball. The kinetic energy of the ball as it leaves the toy is 0.72 J.
- (i) The toy converts the elastic potential energy of the spring into the kinetic energy of the ball. Use the information in (a)(ii) to calculate the percentage efficiency of this conversion.

efficiency = % [1]

- (ii) Determine the initial momentum of the ball as it leaves the toy.

momentum = Ns [3]

- (c) The ball in (b) leaves the toy at point A and moves vertically upwards through the air. Point B is the position of the ball when it is at maximum height h above point A, as illustrated in Fig. 3.2.

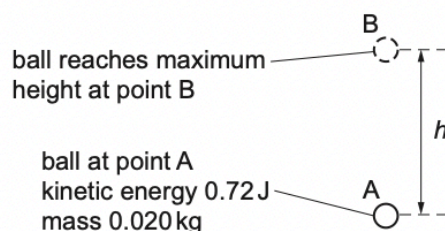


Fig. 3.2 (not to scale)

The gravitational potential energy of the ball increases by 0.60 J as it moves from A to B.

(i) Calculate h .

$h = \dots\dots\dots$ m [2]

(ii) Determine the average force due to air resistance acting on the ball for its movement from A to B.

average force = $\dots\dots\dots$ N [2]

(iii) When there is air resistance, the ball takes time T to move from A to B.

State and explain whether the time taken for the ball to move from A to its maximum height will be more than, less than or equal to time T if there is **no** air resistance.

$\dots\dots\dots$
 $\dots\dots\dots$ [1]

[Total: 13]

- 22 (a) Fig. 2.1 shows the velocity–time graph for an object moving in a straight line.

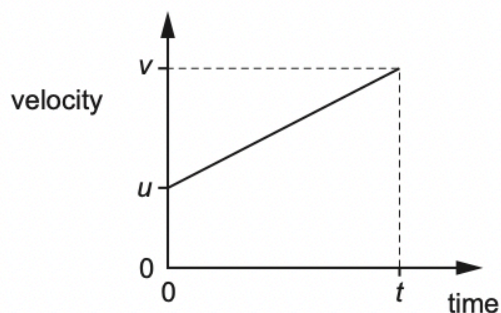


Fig. 2.1

- (i) Determine an expression, in terms of u , v and t , for the area under the graph.

area = [1]

- (ii) State the name of the quantity represented by the area under the graph.

..... [1]

- (b) A ball is kicked with a velocity of 15 m s^{-1} at an angle of 60° to horizontal ground. The ball then strikes a vertical wall at the instant when the path of the ball becomes horizontal, as shown in Fig. 2.2.

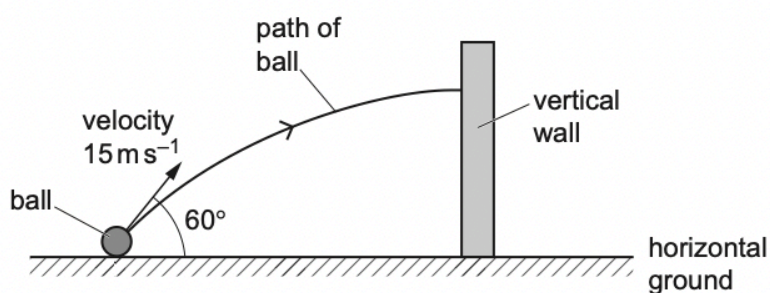


Fig. 2.2 (not to scale)

Assume that air resistance is negligible.

- (i) By considering the vertical motion of the ball, calculate the time it takes to reach the wall.

time = s [3]

- (ii) Explain why the horizontal component of the velocity of the ball remains constant as it moves to the wall.

.....
..... [1]

- (iii) Show that the ball strikes the wall with a horizontal velocity of 7.5 ms^{-1} .

[1]

- (c) The mass of the ball in (b) is 0.40 kg . It is in contact with the wall for a time of 0.12 s and rebounds horizontally with a speed of 4.3 ms^{-1} .

- (i) Use the information from (b)(iii) to calculate the change in momentum of the ball due to the collision.

change in momentum = kg ms^{-1} [2]

- (ii) Calculate the magnitude of the average force exerted on the ball by the wall.

average force = N [1]

[Total: 10]

- 23 A small remote-controlled model aircraft has two propellers, each of diameter 16 cm. Fig. 3.1 is a side view of the aircraft when hovering.

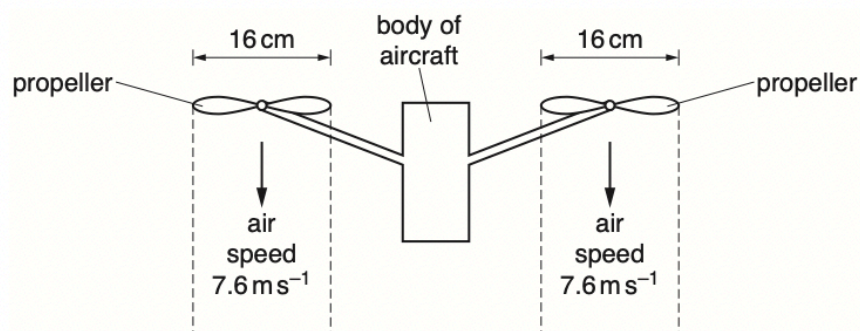


Fig. 3.1

Air is propelled vertically downwards by each propeller so that the aircraft hovers at a fixed position. The density of the air is 1.2 kg m^{-3} . Assume that the air from each propeller moves with a constant speed of 7.6 m s^{-1} in a uniform cylinder of diameter 16 cm. Also assume that the air above each propeller is stationary.

- (a) Show that, in a time interval of 3.0 s, the mass of air propelled downwards by **one** propeller is 0.55 kg.

[3]

- (b) Calculate:

- (i) the increase in momentum of the mass of air in (a)

increase in momentum = N s [1]

- (ii) the downward force exerted on this mass of air by the propeller.

force = N [1]

(c) State:

(i) the upward force acting on **one** propeller

force = N [1]

(ii) the name of the law that explains the relationship between the force in (b)(ii) and the force in (c)(i).

..... [1]

(d) Determine the mass of the aircraft.

mass = kg [1]

(e) In order for the aircraft to hover at a very high altitude (height), the propellers must propel the air downwards with a greater speed than when the aircraft hovers at a low altitude. Suggest the reason for this.

.....

..... [1]

1 i)	The total kinetic energy changes (before and after the collision) so the collision is inelastic OR (relative) speed of approach does not equal (relative) speed of separation so the collision is inelastic	B1 (B1)	Allow kinetic energy of the ball changes/decreases Not kinetic energy of the ball increases Allow relative speed of ball to the ground has changed/decreased so the collision is inelastic Not relative speed of the ball has increased Ignore just 'speed changes' (as speed is changing throughout collision)
4(b)(i)	$p = mv$ or 0.25×3.6 or 0.25×5.2 $\Delta p = 0.25 \times (3.6 + 5.2)$ $= 2.2 \text{ kg m s}^{-1}$	C1 A1	Allow -2.2 kg m s^{-1}
4(b)(ii)	$F = (\Delta p)/t$ $= 2.2 / 0.18$ $= 12 \text{ N}$ Or $F = ma$ and $a = (v-u)/t$ $= m\Delta v/t$ $= 0.25 \times (3.6 + 5.2) / 0.18$ $= 12 \text{ N}$	C1 A1 (C1) (A1)	ECF change in momentum from b(i) ECF change in velocity from b(i)
4(c)	$\frac{1}{2} mv^2 = mg(\Delta)h$ $h_2/h_1 = 3.6^2 / 5.2^2$ $h_2/h_1 = 0.48$ OR $s = v^2/2a$ $h_1 = 5.2^2/2g (= 1.4 \text{ m})$ or $h_2 = 3.6^2/2g (= 0.66 \text{ m})$ $h_2/h_1 = 0.48$	C1 C1 A1 (C1) (C1) (A1)	Any subject

2 a	<u>sum/total</u> momentum (of a system of bodies) is constant or <u>sum/total</u> momentum before = <u>sum/total</u> momentum after for an isolated system / no (resultant) <u>external</u> force	M1 A1	Allow 'conserved' instead of 'constant' Allow M1 mark if 'sum/total' stated once only. Allow 'constant' to mean 'the same' e.g. the total momentum is constant before and after the collision'. if does refer to bodies it must be plural, so not 'total momentum of (a/the) body before = total momentum of (a/the) body after' which is M0 and 0/2.
4bi	$p = mv$ $(4.0 \times 6.0) + (2.0 \times 3.0) = 6.0 \times v$ so $v = 5.0 \text{ m s}^{-1}$	C1 A1	Allow 5 as the number is exact.
4bii	KE before $= (0.5 \times 4.0 \times 6.0^2) + (0.5 \times 2.0 \times 3.0^2)$ $= 81 \text{ J}$ KE after $= (0.5 \times 6.0 \times 5.0^2)$ $= 75 \text{ J}$ percentage transferred $= [(81 - 75) / 81] \times 100$ $= 7\%$	C1 (C1) A1	Allow either numerical expression of kinetic energy for the C mark. Allow ecf of v from (b)(i) (7.4%)

3 a)	product of mass and velocity	B1	allow 'mass × velocity' not 'mass × speed' ignore just 'mv' unless explains 'm' and 'v' ignore any units given
2(b)	change in momentum = $(-1.4) - (+2.8)$ = $(-) 4.2 \text{ kg ms}^{-1}$	A1	allow positive or negative answer
2(c)	$F = \Delta p / (\Delta)t$ or $F = \text{gradient}$ = $4.2 / 12$ = 0.35 N	C1 A1	allow $F = \Delta mv / (\Delta)t$ allow any subject for the symbol equation. ECF value of Δp from (b). Candidates may choose different points on the line eg. $F = 2.8 / 8.0 = 0.35 \text{ N}$. allow negative answer
2(d)	constant/uniform (rate of) decrease (of speed to zero).	B1	accept 'linear decrease (of speed to zero)'. not 'constant deceleration (to zero)' because this does not directly describe the speed.
2(e)	<ul style="list-style-type: none"> the force is constant / does not decrease air resistance would vary / decrease the force is not zero when speed/velocity is zero / at 8 s Any two of the above 3 marking points (1 mark each, max 2)	B2	allow 'resistive force / drag / viscous force / frictional force' instead of 'air resistance' allow 'momentum' instead of speed/velocity
2(f)	line from origin with decreasing positive gradient line has zero gradient at $t = 8.0 \text{ s}$ after $t = 8.0 \text{ s}$ the line has an increasing negative gradient <u>and</u> a positive value of d at $t = 12 \text{ s}$	B1 B1 B1	allow $t = 8.0 \pm 0.4 \text{ s}$

4	$a = (v^2 - u^2) / 2s$ $= (22^2 - 13^2) / (2 \times 180)$ $= 0.88 \text{ ms}^{-2}$	C1
	OR [$t = (180 \times 2) / (22 + 13) = 10.3$] $180 = 13 \times 10.3 + \frac{1}{2} a \times 10.3^2$ or $180 = 22 \times 10.3 - \frac{1}{2} a \times 10.3^2$ or $22 = 13 + a \times 10.3$ $a = 0.88 \text{ ms}^{-2}$	(C1) (A1)
3(a)(ii)	$(\Delta)E = \frac{1}{2}m(\Delta)v^2$	C1
	gain in KE = $\frac{1}{2}m(v^2 - u^2)$ $= \frac{1}{2} \times 9400 \times (22^2 - 13^2)$ $= 1.5 \times 10^6 \text{ J}$	C1 A1
	OR $W = Fs$ $= ma \times d$ gain in KE = $9400 \times 0.88 \times 180$ $= 1.5 \times 10^6 \text{ J}$	(C1) (A1)
3(b)(i)	rate of change of momentum	B1
3(b)(ii)	(Force \Rightarrow) $(2.5 \times 10^4 - 21 \times 10^4) / 15 = -1.2 \times 10^4 \text{ (N)}$	A1

5 a)	product of mass and velocity	B1
2(b)(i)	maximum speed $= 7.2 \times 10^4 / 1800$ $= 40 \text{ m s}^{-1}$	A1
2(b)(ii)	kinetic energy $= \frac{1}{2}mv^2$ $= \frac{1}{2} \times 1800 \times 40^2$ $= 1.4 \times 10^6 \text{ J}$	C1 A1
2(b)(iii)	$a = \text{gradient of line / mass}$ or $a = (v - u) / t$ or $a = \Delta v / t$ $a = \text{e.g. } (3.6 \times 10^4) / (4.0 \times 1800) = 5.0 \text{ m s}^{-2}$ or $a = \text{e.g. } 20 (-0) / 4 = 5.0 \text{ m s}^{-2}$ or $F = \text{e.g. } 3.6 \times 10^4 / 4.0 = 9.0 \times 10^3$ and $a = 9.0 \times 10^3 / 1800 = 5.0 \text{ m s}^{-2}$	C1 A1
2(b)(iv)	distance $= \frac{1}{2} \times 40 \times (8 + 4)$ or distance $= \frac{1}{2} \times 72\,000 \times (8 + 4) / 1800$ or distance $= [40^2 / (2 \times 5)] + [40^2 / (2 \times 10)]$ or distance $= \frac{1}{2} \times 5 \times 8^2 + (40 \times 4 - \frac{1}{2} \times 10 \times 4^2)$ distance $= 240 \text{ m}$	C1 A1
2(c)	stepped shape, showing one constant value up to 8.0 s then stepping to a different constant value from 8.0 s with no time delay	B1
	horizontal straight line from (0, 5.0) to (8.0, 5.0)	B1
	horizontal straight line from (8.0, -10.0) to (12.0, -10.0)	B1

6 a)	<u>sum / total</u> momentum (of a system of bodies) is constant or <u>sum / total</u> momentum before = <u>sum / total</u> momentum after for an isolated system / no (resultant) <u>external</u> force	M1 A1
2(b)(i)	$240 \times 16 = 480v$ and so $v = 8.0 \text{ m s}^{-1}$ or (initial momentum =) $240 \times 16 (= 3840 \text{ g m s}^{-1})$ and $v = 3840 / 480 = 8.0 \text{ m s}^{-1}$	A1
2(b)(ii)	$(E_K =) \frac{1}{2}mv^2$ $\Delta E_K = \frac{1}{2}[(0.24 \times 16^2) - (0.48 \times 8.0^2)]$ $= 15 \text{ J}$	C1 C1 A1
2(c)(i)	$F = (0.24 \times 16) / (2.0 \times 10^{-3})$ or $F = (0.48 \times 8) / (2.0 \times 10^{-3})$ $= 1900 \text{ N}$ direction: to the left	C1 A1 B1
2(c)(ii)	equal (magnitude) opposite (direction)	B1 B1

7 a)	product of mass and velocity	B1
2(b)	$F = m\Delta v / \Delta t$ $= 16 \times 0.60 / 1.1$	C1
	$= 8.7 \text{ N}$	A1
2(c)	$x = ut + \frac{1}{2}at^2$ $x = 0.60 \times 3.7 + \frac{1}{2} \times 0.85 \times 3.7^2$	C1
	or	
	$v = 0.60 + 0.85 \times 3.7 (= 3.75 \text{ m s}^{-1})$ $x = 3.75 \times 3.7 - 0.5 \times 0.85 \times 3.7^2$ or $x = \frac{1}{2} \times (0.60 + 3.75) \times 3.7$ or $x = (3.75^2 - 0.60^2) / (2 \times 0.85)$	(C1)
	$x = 8.0 \text{ m}$	A1
2(d)(i)	$F = W / s$	C1
	$= 250 / 18$	A1
	$= 14 \text{ N}$	
2(d)(ii)	any line starting at distance = 0 and a positive non-zero value of kinetic energy	B1
	a straight line from distance = 0 to distance = x with positive gradient	B1
	a straight horizontal line at a non-zero value of kinetic energy starting at distance = x and ending at distance = x + 18 m that is continuous with the previous line	B1

8 a)	<u>sum / total</u> momentum before (a collision) = <u>sum / total</u> momentum after (a collision) or <u>sum / total</u> momentum (of a system) is constant	M1
	if no (resultant) external force (acts) / for an isolated system	A1
3(b)	along direction of motion: $10m = 2mv \cos 30^\circ + 3mw \cos 30^\circ$	C1
	perpendicular to direction of motion: $2mv \cos 60^\circ = 3mw \cos 60^\circ$ $(v = 3w / 2)$	C1
	$v = 2.9 \text{ m s}^{-1}$	A1
	$w = 1.9 \text{ m s}^{-1}$	A1
3(c)	$E_k = \frac{1}{2} \times m \times v^2$ $(= \frac{1}{2} \times 4.2 \times 6.0^2)$ $(= 76 \text{ J})$	C1
	force = work done / distance	C1
	force = $76 / 0.050$ $= 1500 \text{ N}$	A1

9 c)	or	
	$a = (-)u^2 / 2s$ $= (-)6.0^2 / (2 \times 0.050)$ $(= (-)360 \text{ m s}^{-2})$	(C1)
	$F = ma$	(C1)
	$F = 4.2 \times 360$ $= 1500 \text{ N}$	(A1)
	or	
	$a = (-)u^2 / 2s$ $= (-)6.0^2 / (2 \times 0.050)$ $(= (-)360 \text{ m s}^{-2})$	(C1)
	$t = -u / a$ $= -6.0 / -360 (= 0.017 \text{ s})$ $F = \Delta p / t$ $F = (0 - 4.2 \times 6) / 0.017$ $= 1500 \text{ N}$	(C1) (A1)

10 a)	displacement	A1
3(a)(i)	$a = \text{gradient or } a = \Delta v / (\Delta t) \text{ or } a = (v - u) / t$	C1
	e.g. $a = (0.30 - 0.12) / (0.35 - 0.15)$	A1
	$a = 0.90 \text{ m s}^{-2}$	
3(b)(ii)	$(0.25 \times 0.48) + (0.75 \times 0.12) = (0.25 v) + (0.75 \times 0.30)$ or $(0.48 - 0.12) = (0.30 - v)$ or $(\frac{1}{2} \times 0.25 \times 0.48^2) + (\frac{1}{2} \times 0.75 \times 0.12^2) = (\frac{1}{2} \times 0.25 \times v^2) + (\frac{1}{2} \times 0.75 \times 0.30^2)$	C1
	$v = (-)0.060 \text{ m s}^{-1}$	A1
	direction: to the left / from the right / opposite to (its) initial velocity / opposite to (initial / final) velocity of B	B1
3(c)	sketch: horizontal line from (0, 0.48) to (0.15, 0.48)	B1
	horizontal line from (0.35, -0.06) to (0.5, -0.06)	B1
	straight line between (0.15, 0.48) and (0.35, -0.06)	B1

11 i)(i)	component of momentum $= 0.25 \times 3.7 \times \sin 27^\circ$	C1
	$= 0.42 \text{ N s}$	A1
4(a)(ii)	$m_{(Z)} \times 5.5 \times \sin 44^\circ = 0.42$ or $m_{(Z)} \times 5.5 \times \sin 44^\circ = 0.25 \times 3.7 \times \sin 27^\circ$ $m_Z = 0.11 \text{ kg}$	A1
4(a)(iii)	magnitudes: equal	B1
	directions: opposite	B1
4(b)	$4 + 6 = 2 + v$ $v = 8 \text{ m s}^{-1}$	A1

12	3(a)	(resultant) force (on an object) is proportional to / equal to the rate of change of momentum	B1
	3(b)(i)	resultant force = e.g. $6.0 / 4.0$ = 1.5 N	A1
	3(b)(ii)	force $X = 1.5 + 2.0$ = 3.5 N	A1
	3(c)	from $t = 0$ to $t = 4.0$ s: horizontal line at any non-zero value of X	B1
		from $t = 0$ to $t = 4.0$ s: horizontal line at $X = 3.5$ N	B1
		from $t = 4.0$ s to $t = 6.0$ s: horizontal line at $X = 2.0$ N	B1

13	a)(i)	rate of change of momentum	B1
	3(a)(ii)	change in momentum = $(1.4 - 0.80) \times 3.0$ = 1.8 kg m s ⁻¹	C1 A1
	3(b)(i)	resultant force (on block) is zero (so) velocity is constant	B1 B1
	3(b)(ii)	$P = Fv$ or $P = Fs / t$ $v = 2.0 / 0.80$ (= 2.5 m s ⁻¹) distance = 2.5×3.0 = 7.5 m or $P = W / t$ or $P = Fs / t$ $W = 2.0 \times 3.0$ (= 6.0 J) distance = $6.0 / 0.80$ = 7.5 m	C1 C1 A1 (C1) (C1) (A1)
	3(c)	0 to 3.0 s: upward sloping straight line from the origin. 3.0 to 6.0 s: horizontal line at non-zero value of momentum with no 'step change' in momentum at 3.0 s	B1 B1

14	a)	<u>sum / total</u> momentum before = <u>sum / total</u> momentum after or <u>sum / total</u> momentum (of a system of objects) is constant if no (resultant) external force / for an isolated system	M1 A1
	3(b)(i)	$3m \times 4 = m \times v \sin \theta$ ($v \sin \theta = 12$) $2m \times 6 = m \times v \cos \theta$ ($v \cos \theta = 12$) therefore $\sin \theta = \cos \theta$ or $\tan \theta = 1$ $\theta = 45^\circ$	C1 C1 A1
	3(b)(ii)	$mv \times \cos 45^\circ = 12m$ or $mv \times \sin 45^\circ = 12m$ or $(mv)^2 = (3m \times 4)^2 + (2m \times 6)^2$ $v = 17 \text{ m s}^{-1}$	C1 A1
	3(c)(i)	(chemical energy) = $0.0050 \times 700 = 3.5$ (J) or (chemical energy) = $5.0 \times 0.700 = 3.5$ (J)	A1
	3(c)(ii)	$E = \frac{1}{2}mv^2$ total $E = (0.5 \times 3m \times 4^2) + (0.5 \times 2m \times 6^2) + (0.5 \times m \times 17^2)$ $3.5 = 204m$ $m = 0.017 \text{ kg}$	C1 C1 A1

15 (a)	$E = \frac{1}{2}mv^2$	C1
	$p = mv$	C1
	$m = 0.37^2 / (2 \times 0.30)$ or $0.37 / 1.6$ or $(0.30 \times 2) / 1.6^2$ $= 0.23 \text{ kg}$	A1
4(b)	$0.37 - 0.65 = -0.13 - p$ $p = 0.15 \text{ kg m s}^{-1}$	A1
4(c)	$7.7 = (0.13 + 0.37) / (\Delta)t$ or $7.7 = (0.65 - 0.15) / (\Delta)t$	C1
	time = 0.065 s	A1

16 (a)	sum/total momentum before = sum/total momentum after or sum/total momentum (of a system of objects) is constant if no (resultant) external force/for a closed system	M1
		A1
4(b)(i)	$(3.0 \times 4.0 \times \cos \theta)$ or $(2.5 \times 4.8 \times \cos \theta)$ or (5.5×3.7)	C1
	$(3.0 \times 4.0 \times \cos \theta) + (2.5 \times 4.8 \times \cos \theta) = (5.5 \times 3.7)$	C1
	$\theta = 32^\circ$	A1
4(b)(ii)	(initial $E_K = \frac{1}{2} \times 3.0 \times 4.0^2 + \frac{1}{2} \times 2.5 \times 4.8^2 = 53 \text{ (J)}$) or (final $E_K = \frac{1}{2} \times 5.5 \times 3.7^2 = 38 \text{ (J)}$)	C1
	values of initial E_K and final E_K both correct and inelastic stated	A1

17 (a)	$E = \frac{1}{2}mv^2$	C1
	$p = mv$	C1
	$m = 0.37^2 / (2 \times 0.30)$ or $0.37 / 1.6$ or $(0.30 \times 2) / 1.6^2$ $= 0.23 \text{ kg}$	A1
4(b)	$0.37 - 0.65 = -0.13 - p$ $p = 0.15 \text{ kg m s}^{-1}$	A1
4(c)	$7.7 = (0.13 + 0.37) / (\Delta)t$ or $7.7 = (0.65 - 0.15) / (\Delta)t$	C1
	time = 0.065 s	A1

18	(a)	mass \times velocity	B1
	2(b)(i)	kinetic energy $= \frac{1}{2}mv^2$	C1
		$= \frac{1}{2} \times 0.24 \times 2.3^2$	C1
		$= 0.63 \text{ J}$	A1
	2(b)(ii)	change in momentum $= \frac{1}{2} \times 240 \times 5.0 \times 10^{-3}$	C1
		$= 0.60 \text{ N s}$	A1
	2(b)(iii)	(change in velocity of Y) $= 0.60 / 0.12$ ($= 5.0 \text{ m s}^{-1}$)	C1
		final velocity of Y $= 5.0 - 2.3$ $= 2.7 \text{ m s}^{-1}$	A1
		or	
		(final momentum of Y) $= 0.60 - 0.12 \times 2.3$ ($= 0.324 \text{ N s}$)	(C1)
		final velocity of Y $= 0.324 / 0.12$ $= 2.7 \text{ m s}^{-1}$	(A1)
	2(c)	sloping straight line from (0, 0) to $t = 3.0 \text{ ms}$ and another straight line continuous with the first from $t = 3.0 \text{ ms}$ to (5.0, 0)	B1
		lines showing maximum force of magnitude 240 N	B1
		lines wholly in the negative F region of the graph	B1

19	3(a)	solid straight line drawn between centre of sphere at X and at Y	B1
	3(b)	$p = mv$ or $0.72 = mv$	C1
		$E = \frac{1}{2}mv^2$ or $0.86 = \frac{1}{2}mv^2$	C1
		($m = 0.72^2 / (2 \times 0.86) = 0.30 \text{ (kg)}$) or $v = 2E_K / p$ $v = (0.86 \times 2) / 0.72 = 2.4 \text{ (to 2 s.f.)}$ $m = 0.72 / 2.4 = 0.30 \text{ (kg)}$	A1
	3(c)	$(\Delta)E = mg(\Delta)h$	C1
		$h = 0.86 / (0.30 \times 9.81)$ $= 0.29 \text{ m}$	A1
	3(d)	$\cos \theta = (0.93 - 0.29) / 0.93$ so $\theta = 47^\circ$	A1
	3(e)	moment $= (0.30 \times 9.81) \times 0.93 \times (\sin 47^\circ \text{ or } \cos 43^\circ)$ or moment $= (0.30 \times 9.81) \times [0.93^2 - (0.93 - 0.29)^2]^{0.5}$ $= 2.0 \text{ N m}$	C1 A1

20 (a)	$v^2 = u^2 + 2as$	C1
	$u^2 = 8.7^2 - (2 \times 9.81 \times 1.5)$	
	$u = 6.8 \text{ m s}^{-1}$	A1
2(b)	(magnitude of) force on ball (by ground) equal to force on ground (by ball)	B1
	(direction of) force on ball (by ground) opposite to force on ground (by ball)	B1
2(c)(i)	$(p =) 0.059 \times 8.7 \text{ or } 0.059 \times 5.4$	C1
	change in momentum = $0.059 (8.7 + 5.4)$	A1
	$= 0.83 \text{ N s}$	
2(c)(ii)	resultant force = $0.83 / 0.091 \text{ or } 0.059 [(8.7 + 5.4) / 0.091]$ $= 9.1 \text{ N}$	A1
2(c)(iii)	$(W =) 0.059 \times 9.81$	C1
	$(W =) 0.58 \text{ (N)}$	A1
	force = $9.1 + 0.58$	
	$= 9.7 \text{ N}$	
2(d)	straight line with a positive gradient and starting from a non-zero value of speed at $t = 0$ and ending when $t = T$	B1
2(e)	air resistance increases	B1
	resultant force/acceleration decreases so gradient (of curve) decreases	B1

21 (a)	(force =) rate of change of momentum	B1
3(b)(i)	$E = \frac{1}{2}mv^2 \text{ or } \frac{1}{2} \times 0.062 \times 3.8^2 \text{ or } \frac{1}{2} \times 0.062 \times 1.7^2$	C1
	loss of KE = $\frac{1}{2} \times 0.062 \times (3.8^2 - 1.7^2)$	A1
	$= 0.36 \text{ J}$	
3(b)(ii)	$p = mv \text{ or } 0.062 \times 3.8 \text{ or } 0.062 \times 1.7$	C1
	change in momentum = $0.062 \times (1.7 + 3.8)$	A1
	$= 0.34 \text{ N s}$	
3(b)(iii)	(average resultant force =) $0.34 / 0.081 = 4.2 \text{ (N)}$ or (average resultant force =) $0.062 \times (1.7 + 3.8) / 0.081 = 4.2 \text{ (N)}$	A1
3(b)(iv)	1. average force = $4.2 + (0.062 \times 9.81)$ $= 4.8 \text{ N}$	A1
	2. average force = 4.8 N	A1

22	3(a)(i)	$F = kx$	C1
		$F_1 = 800 \times 0.045$ $= 36 \text{ N}$	A1
	3(a)(ii)	$(E =) \frac{1}{2}kx^2$ or $\frac{1}{2}Fx$ or area under graph	C1
		$\frac{1}{2} \times 800 \times (0.045)^2$ or $\frac{1}{2} \times 36 \times 0.045 = 0.81 \text{ (J)}$	A1
	3(b)(i)	efficiency $= (0.72 / 0.81) \times 100$ $= 89\%$	A1
	3(b)(ii)	$E = \frac{1}{2}mv^2$	C1
		$p = mv$	C1
		$0.72 = \frac{1}{2} \times 0.020 \times v^2$ and $p = 0.020 \times v$	A1
		$p = 0.17 \text{ N s}$	
	3(c)(i)	$(\Delta)E = mg(\Delta)h$	C1
		$h = 0.60 / (0.020 \times 9.81) = 3.1 \text{ m}$	A1
	3(c)(ii)	$F = (0.72 - 0.60) / 3.1$	C1
		$= 0.039 \text{ N}$	A1
	3(c)(iii)	resultant force on ball is less (than that with air resistance) so time (taken) is more (than T)	B1

23	2(a)(i)	area $= ut + \frac{1}{2}(v - u)t$ or area $= vt - \frac{1}{2}(v - u)t$ or area $= \frac{1}{2}(u + v)t$	A1
	2(a)(ii)	displacement	A1
	2(b)(i)	$u = 15 \sin 60^\circ (= 13 \text{ m s}^{-1})$	C1
		$t = 15 \sin 60^\circ / 9.81$	C1
		$= 1.3 \text{ s}$	A1
	2(b)(ii)	the force in the horizontal direction is zero	B1
	2(b)(iii)	(velocity $=$) $15 \cos 60^\circ = 7.5 \text{ (m s}^{-1}\text{)}$ or (velocity $=$) $15 \sin 30^\circ = 7.5 \text{ (m s}^{-1}\text{)}$	A1
	2(c)(i)	$p = mv$ or 0.40×7.5 or 0.40×4.3	C1
		$\Delta p = 0.40 (7.5 + 4.3)$	A1
		$= 4.7 \text{ kg m s}^{-1}$	
	2(c)(ii)	force $= 4.7 / 0.12$ or $0.40 \times [(7.5 + 4.3) / 0.12]$ $= 39 \text{ N}$	A1

20	(a)	$\rho = m / V$	C1
		$V = \pi \times (0.16 / 2)^2 \times 7.6 \times 3.0 \text{ (= 0.458 m}^3\text{)}$	C1
		$m = \pi \times (0.16 / 2)^2 \times 7.6 \times 3.0 \times 1.2 = 0.55 \text{ kg}$	A1
	3(b)(i)	$\Delta p = 0.55 \times 7.6$ $= 4.2 \text{ N s}$	A1
	3(b)(ii)	$F = 4.2 / 3.0 \text{ or } 0.55 \times 7.6 / 3.0$ $= 1.4 \text{ N}$	A1
	3(c)(i)	$F = 1.4 \text{ N}$	A1
	3(c)(ii)	Newton's third law (of motion)	B1
	3(d)	$2 \times 1.4 = m \times 9.81$ $m = 0.29 \text{ kg}$	A1
	3(e)	the density of air is less at high altitude	B1
	3(f)	$f_o = f_s v / (v - v_s)$ $= 3000 \times 340 / (340 - 22)$	C1
		$= 3200 \text{ Hz}$	A1